Practice problems for Final.

1. Given $\vec{a} =< 1, 1, 2 >$ and $\vec{b} =< 2, -1, 0 >$. Find the area of the parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$.

2. Find an equation of the line through the point $(1, 2, -1)$ and perpendicular to the plane
$$2x + y + z = 2$$

3. Find the distance from the point $(1, -1, 2)$ to the plane
$$x + 3y + z = 7$$

4. Find an equation of the plane that passes through the point $(-1, -3, 1)$ and contains the line $x = -1 - 2t$, $y = 4t$, $z = 2 + t$.

5. Find parametric equations of the line of intersection of the planes $z = x+y$ and $2x-5y-z = 1$.

6. Are the lines $x = -1 + 4t$, $y = 3 + t$, $z = 1$ and $x = 13 - 8s$, $y = 1 - 2s$, $z = 2$ parallel, skew or intersecting? If they intersect, find the point of intersection.

7. Identify and roughly sketch the following surfaces. Find traces in the planes $x = k$, $y = k$, $z = k$
   (a) $4x^2 + 9y^2 + 36z^2 = 36$
   (b) $y = x^2 + z^2$
   (c) $4z^2 - x^2 - y^2 = 1$
   (d) $x^2 + 2z^2 = 1$

8. Find
$$\lim_{t\to 1} \left(\sqrt{t+3}\vec{\imath} + \frac{t-1}{t^2-1}\vec{\jmath} + \frac{\tan t}{t}\vec{k} \right)$$

9. Find the unit tangent vector $\vec{T}(t)$ for the vector function $\vec{r}(t) =< t, 2\sin t, 3\cos t >$.

10. Evaluate
$$\int_1^4 \left(\sqrt{t}\vec{\imath} + te^{-t}\vec{\jmath} + \frac{1}{t^2}\vec{k} \right) dt$$

11. Find the length of the curve given by the vector function $\vec{r}(t) = \cos^3 t\ \vec{\imath} + \sin^3 t\ \vec{\jmath} + \cos(2t)\ \vec{k}$, $0 \le t \le \frac{\pi}{2}$.

12. Find the curvature of the curve $\vec{r}(t) =< 2t^3, -3t^2, 6t >$.

13. Find an equation of the normal plane to the curve $\vec{r}(t) =< t^2, 2t, \ln t >$ at the point where $t = 1$.

14. Sketch the domain of the function
$$f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2)$$

15. Find the level curves of the function $z = x - y^2$.

16. Find f_{xyz} if $f(x, y, z) = e^{xyz}$.

17. The dimensions of a closed rectangular box are 80 cm, 60 cm, and 50 cm with a possible error of 0.2 cm in each dimension. Use differential to estimate the maximum error in surface area of the box.

18. Find parametric equations of the normal line and an equation of the tangent plane to the surface

$$x^3 + y^3 + z^3 = 5xyz$$

at the point $(2, 1, 1)$.

19. Given that $w(x, y) = 2\ln(3x + 5y) + x - 2\tan^{-1} y$, where $x = s - \cot t$, $y = s + \sin^{-1} t$. Find $\dfrac{\partial w}{\partial t}$.

20. Let $f(x, y, z) = \ln(2x + 3y + 6z)$. Find a unit vector in the direction in which f decreases most rapidly at the point $P(-1, -1, 1)$ and find the derivative (rate of change) of f in this direction.

21. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if

$$xe^y + yz + ze^x = 0$$

22. Find the local extrema/saddle points for

$$f(x, y) = 2x^2 + y^2 + 2xy + 2x + 2y$$

23. Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + 2xy + 3y^2$ over the set D, where D is the closed triangular region with vertices $(-1, 1)$, $(2, 1)$, and $(-1, -2)$.

24. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = xy$ subject to the constraint $9x^2 + y^2 = 4$.

25. Sketch the region of integration and change the order of integration for $\int_0^1 \int_{y^2}^{2-y} f(x, y)dxdy$.

26. Evaluate $\iint_D (xy+2x+3y)dA$, where D is the region in the first quadrant bounded by $x = 1-y^2$, $x = 0$, $y = 0$.

27. Sketch the region whose area is given by the integral $\int_0^\pi \int_1^{1+\sin\theta} rdrd\theta$.

28. Find the area inside one petal of the rose $r = 2\sin(2\theta)$ outside the circle $r = 1$. Sketch the region of integration.

29. Find the mass and center of mass of a lamina that occupies the region D bounded by the lines $y = 0$, $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$ that lies in the first quadrant if the density function is $\rho(x, y) = xy^2$.

30. Evaluate $\iiint_E (x + 2y)dV$ if E is bounded by the cylinder $x = y^2$ and the planes $z = 0$ and $x + z = 1$.

31. Sketch the solid whose volume is given by the integral $\int_1^3 \int_0^{\pi/2} \int_r^3 rdzd\theta dr$

32. Evaluate $\iiint_E \sqrt{x^2+y^2}dV$, where E is the solid bounded by the paraboloid $z = 9 - x^2 - y^2$ and the xy-plane.

33. Sketch the solid whose volume is given by the integral $\int_0^{2\pi} \int_0^{\pi/6} \int_1^3 \rho^2 \sin\varphi d\rho d\varphi d\theta$

34. Evaluate $\iiint_E xe^{(x^2+y^2+z^2)^2} dV$ if the E is the solid that lies between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$ in the first octant.

35. Find the gradient vector field of the function $f(x, y, z) = xy^2 - yz^3$.

36. Evaluate the line integral $\int_C x^3 z ds$ if C is given by $x = 2\sin t$, $y = t$, $z = 2\cos t$, $0 \le t \le \pi/2$.

37. Evaluate $\int_C ydx + zdy + xdz$ if C consists of the line segments from (0,0,0) to (1,1,2) and from (1,1,2) to (3,1,4).

38. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = x^2y\vec{\imath} + e^y\vec{\jmath}$ and C is given by $\vec{r}(t) = t^2\vec{\imath} - t^3\vec{\jmath}$, $0 \le t \le 1$.

39. Show that $\vec{F}(x, y) = (2x + y^2 + 3x^2y)\vec{\imath} + (2xy + x^3 + 3y^2)\vec{\jmath}$ is conservative vector field. Use this fact to evaluate $\int_C \vec{F} \cdot d\vec{r}$ if C is the arc of the curve $y = x\sin x$ from (0,0) to $(\pi, 0)$.

40. Show that $\vec{F}(x, y, z) = yz(2x + y)\vec{\imath} + xz(x + 2y)\vec{\jmath} + xy(x + y)\vec{k}$ is conservative vector field. Use this fact to evaluate $\int_C \vec{F} \cdot d\vec{r}$ if C is given by $\vec{r}(t) = (1+t)\vec{\imath} + (1+2t^2)\vec{\jmath} + (1+3t^3)\vec{k}$, $0 \le t \le 1$.

41. Use Green's Theorem to evaluate $\int_C x^2ydx - xy^2dy$ where C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.

42. Find curl $\vec{F}$ and div $\vec{F}$ if $\vec{F} = x^2z\vec{\imath} + 2x\sin y\vec{\jmath} + 2z\cos y\vec{k}$.

43. Find an equation of the tangent plane to the surface given by parametric equations $x = u^2$, $y = u - v^2$, $z = v^2$, at the point (1,0,1).

44. Find the area of the hyperbolic paraboloid $z = x^2 - y^2$ that lies between the cylinders $x^2+y^2 = 1$ and $x^2 + y^2 = 4$.

45. Find the area of the surface with parametric equations $x = uv$, $y = u+v$, $z = u-v$, $u^2+v^2 \le 1$.

46. Find the mass of a thin funnel in the shape of a cone $z = \sqrt{x^2 + y^2}$, $1 \le z \le 4$ if its density function is $\rho(x, y, z) = 10 - z$.

47. Evaluate $\iint_S yz\, dS$ if S is the part of the plane $z = y+3$ that lies inside the cylinder $x^2+y^2 = 1$.

48. Let T be the solid bounded by the paraboloids

$$z = x^2 + 2y^2, \text{ and } z = 12 - 2x^2 - y^2.$$

Let $\vec{F} =< x, y, z >$. Find the outward flux of $\vec{F}$ across the boundary surface of T.

49. Verify the Divergence Theorem for $\vec{F} =< x^2, xy, z >$ and the region E bounded by the coordinate planes and the plane $2x + 3y + 4z = 12$.

50. Use Stokes Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F} =< 2z, x, 3y >$ and C is the ellipse in which the plane $z = x$ meets the cylinder $x^2 + y^2 = 4$, oriented counterclockwise as viewed from above.