Practice problems for Exam 1.

1. Given $\vec{a} =< 1, 1, 2 >$ and $\vec{b} =< 2, -1, 0 >$. Find the area of the parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$.

2. Find an equation of the line through the point $(1, 2, -1)$ and perpendicular to the plane
$$2x + y + z = 2$$

3. Find the distance from the point $(1, -1, 2)$ to the plane
$$x + 3y + z = 7$$

4. Find an equation of the plane that passes through the point $(-1, -3, 1)$ and contains the line $x = -1 - 2t$, $y = 4t$, $z = 2 + t$.

5. Find parametric equations of the line of intersection of the planes $z = x+y$ and $2x-5y-z = 1$.

6. Are the lines $x = -1 + 4t$, $y = 3 + t$, $z = 1$ and $x = 13 - 8s$, $y = 1 - 2s$, $z = 2$ parallel, skew or intersecting? If they intersect, find the point of intersection.

7. Identify and roughly sketch the following surfaces. Find traces in the planes $x = k$, $y = k$, $z = k$
   (a) $4x^2 + 9y^2 + 36z^2 = 36$
   (b) $y = x^2 + z^2$
   (c) $4z^2 - x^2 - y^2 = 1$
   (d) $x^2 + 2z^2 = 1$

8. Find
$$\lim_{t\to 1}\left(\sqrt{t+3}\vec{\imath} + \frac{t-1}{t^2-1}\vec{\jmath} + \frac{\tan t}{t}\vec{k}\right)$$

9. Find the unit tangent vector $\vec{T}(t)$ for the vector function $\vec{r}(t) =< t, 2\sin t, 3\cos t >$.

10. Evaluate
$$\int_1^4 \left(\sqrt{t}\vec{\imath} + te^{-t}\vec{\jmath} + \frac{1}{t^2}\vec{k}\right) dt$$

11. Find the length of the curve given by the vector function $\vec{r}(t) = \cos^3 t\ \vec{\imath} + \sin^3 t\ \vec{\jmath} + \cos(2t)\ \vec{k}$, $0 \le t \le \frac{\pi}{2}$.

12. Find the curvature of the curve $\vec{r}(t) =< 2t^3, -3t^2, 6t >$.

13. Find an equation of the normal plane to the curve $\vec{r}(t) =< t^2, 2t, \ln t >$ at the point where $t = 1$.

14. Sketch the domain of the function
$$f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2)$$

15. Find the level curves of the function $z = x - y^2$.

16. Find f_{xyz} if $f(x, y, z) = e^{xyz}$.

17. The dimensions of a closed rectangular box are 80 cm, 60 cm, and 50 cm with a possible error of 0.2 cm in each dimension. Use differential to estimate the maximum error in surface area of the box.

18. Find parametric equations of the normal line and an equation of the tangent plane to the surface

$$x^3 + y^3 + z^3 = 5xyz$$

at the point $(2, 1, 1)$.

19. Given that $w(x, y) = 2\ln(3x + 5y) + x - 2\tan^{-1} y$, where $x = s - \cot t$, $y = s + \sin^{-1} t$. Find $\dfrac{\partial w}{\partial t}$.

20. Let $f(x, y, z) = \ln(2x + 3y + 6z)$. Find a unit vector in the direction in which f decreases most rapidly at the point $P(-1, -1, 1)$ and find the derivative (rate of change) of f in this direction.

21. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if

$$xe^y + yz + ze^x = 0$$

22. Find the local extrema/saddle points for

$$f(x, y) = 2x^2 + y^2 + 2xy + 2x + 2y$$

23. Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + 2xy + 3y^2$ over the set D, where D is the closed triangular region with vertices $(-1, 1)$, $(2, 1)$, and $(-1, -2)$.

24. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = xy$ subject to the constraint $9x^2 + y^2 = 4$.