Practice problems for Exam 2.

1. Sketch the region of integration and change the order of integration for $\int_0^1 \int_{y^2}^{2-y} f(x,y)dxdy$.

2. Evaluate $\iint_D (xy+2x+3y)dA$, where D is the region in the first quadrant bounded by $x = 1-y^2$, $x = 0$, $y = 0$.

3. Sketch the region whose area is given by the integral $\int_0^{\pi} \int_1^{1+\sin\theta} r dr d\theta$.

4. Find the area of the region enclosed by the lemniscate $r^2 = 4\cos(2\theta)$.

5. Find the mass and center of mass of a lamina that occupies the region D bounded by the lines $y = 0$, $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$ that lies in the first quadrant if the density function is $\rho(x,y) = xy^2$.

6. Evaluate $\iiint_E (x + 2y)dV$ if E is bounded by the cylinder $x = y^2$ and the planes $z = 0$ and $x + z = 1$.

7. Sketch the solid whose volume is given by the integral $\int_1^3 \int_0^{\pi/2} \int_r^3 r dz d\theta dr$

8. Evaluate $\iiint_E \sqrt{x^2 + y^2} dV$, where E is the solid bounded by the paraboloid $z = 9 - x^2 - y^2$ and the xy-plane.

9. Sketch the solid whose volume is given by the integral $\int_0^{2\pi} \int_0^{\pi/6} \int_1^3 \rho^2 \sin\varphi d\rho d\varphi d\theta$

10. Evaluate $\iiint_E x e^{(x^2+y^2+z^2)^2} dV$ if the E is the solid that lies between the spheres $x^2+y^2+z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

11. Find the gradient vector field of the function $f(x,y,z) = xy^2 - yz^3$.

12. Evaluate the line integral $\int_C x^3 z ds$ if C is given by $x = 2\sin t$, $y = t$, $z = 2\cos t$, $0 \le t \le \pi/2$.

13. Evaluate $\int_C y dx + z dy + x dz$ if C consists of the line segments from (0,0,0) to (1,1,2) and from (1,1,2) to (3,1,4).

14. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = x^2 y \vec{\imath} + e^y \vec{\jmath}$ and C is given by $\vec{r}(t) = t^2 \vec{\imath} - t^3 \vec{\jmath}$, $0 \le t \le 1$.

15. Show that $\vec{F}(x,y) = (2x + y^2 + 3x^2 y)\vec{\imath} + (2xy + x^3 + 3y^2)\vec{\jmath}$ is conservative vector field. Use this fact to evaluate $\int_C \vec{F} \cdot d\vec{r}$ if C is the arc of the curve $y = x\sin x$ from (0,0) to $(\pi, 0)$.

16. Show that $\vec{F}(x,y,z) = yz(2x + y)\vec{\imath} + xz(x + 2y)\vec{\jmath} + xy(x + y)\vec{k}$ is conservative vector field. Use this fact to evaluate $\int_C \vec{F} \cdot d\vec{r}$ if C is given by $\vec{r}(t) = (1 + t)\vec{\imath} + (1 + 2t^2)\vec{\jmath} + (1 + 3t^3)\vec{k}$, $0 \le t \le 1$.

17. Use Green's Theorem to evaluate $\int_C x^2 y dx - xy^2 dy$ where C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.

18. Find curl $\vec{F}$ and div $\vec{F}$ if $\vec{F} = x^2 z \vec{\imath} + 2x \sin y \vec{\jmath} + 2z \cos y \vec{k}$.

19. Show that there is no vector field $\vec{G}$ such that curl $\vec{G} = 2x\vec{\imath} + 3yz\vec{\jmath} - xz^2\vec{k}$.