The correct solution for Problem 2.

2. Evaluate $\iint_D (xy + 2x + 3y)dA$, where D is the region in the first quadrant bounded by $x = 1 - y^2$, $x = 0$, $y = 0$.

SOLUTION.

$$\iint_D (xy + 2x + 3y)dA = \int_0^1 \int_0^{1-y^2} (xy + 2x + 3y)dxdy = \int_0^1 \left(\frac{x^2}{2}y + x^2 + 3xy \right)\Bigg|_{x=0}^{x=1-y^2} dy$$

$$= \int_0^1 \left(\frac{1}{2}y(1-y^2)^2 + (1-y^2)^2 + 3(1-y^2)y \right) dy = \frac{1}{2}\int_0^1 y(1-y^2)^2 dy + \int_0^1 (1-2y^2+y^4+3y-3y^3)dy$$

Substitution for the first integral: $1 - y^2 = u$, then $du = -2ydy$, $0 \to 1$, $1 \to 0$

$$= -\frac{1}{4}\int_1^0 u^2 du + \left(y - 2\frac{y^3}{3} + \frac{y^5}{5} + 3\frac{y^2}{2} - 3\frac{y^4}{4} \right)\Bigg|_0^1 = -\frac{1}{12}u^3\Bigg|_1^0 + 1 - \frac{2}{3} + \frac{1}{5} + \frac{3}{2} - \frac{3}{4} = \frac{1}{12} + \frac{77}{60} = \frac{41}{30}$$