MATH 251–507, 508, 511

Midterm Exam I, October 7, 2010

Show all your work neatly and concisely and **clearly indicate your final answer**. The use of a calculator, laptop or computer is prohibited.

1. *[10 pts.]* Find an equation of the plane that passes through the points $(-1, 2, 0)$, $(2, 0, 1)$, and $(-5, 3, 1)$.

2. *[13 pts.]* Find the length of the curve $\vec{r}(t) =< 6t, 3\sqrt{2}\, t^2, 2t^3 >$, $0 \leq t \leq 1$.

3. *[13 pts.]* Find the curvature of the curve $\vec{r}(t) =< t^2 + 2, t^2 - 4t, 2t >$.

4. *[12 pts.]* Let $f(x, y, z) = x + \ln(y^2 + z^2)$. Find a vector in the direction in which f increases most rapidly at the point $P(2, 1, 1)$.

5. *[12 pts.]* Find **parametric** equations of the normal line to the surface $xy^2z^3 = 12$ at the point $(3, 2, 1)$.

6. *[12 pts.]* Use the differential to estimate
$$\sqrt{5(1.04)^2 + 4(0.95)^2}$$

7. *[13 pts.]* Given that $w = 5x^3y + x$ where $x = t\tan s$, $y = t + \ln s$, use the Chain Rule to find $\dfrac{\partial w}{\partial s}$.

8. *[15 pts.]* Find the dimensions of the rectangular box with largest volume if the total surface area is given as 64 cm^2.

Bonus Problem (*[10 pts]*, **no partial credit**). The plane $y + z = 3$ intersects the cylinder $x^2 + y^2 = 5$ in an ellipse. Find **symmetric** equations for the tangent line to this ellipse at the point $(1, 2, 1)$.