MATH 251–507, 508, 511

Midterm Exam II, November 18, 2010

Instructions:

- **Show all your work** neatly and concisely and **clearly indicate your final answer**.
- **Do not simplify the final answer.**
- The use of a calculator, laptop or computer is prohibited.

GOOD LUCK!

1. *[11 pts.]* Sketch the region of integration and change the order of integration for $\int_0^1 \int_{2x}^{3x} f(x, y)dydx$.
2. *[16 pts.]* Find the area inside one petal of the rose $r = 2\sin(2\theta)$ outside the circle $r = 1$. Sketch the region of integration.
3. *[17 pts.]* Let E be a solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x+2y+z = 4$. Sketch the solid E and find its volume.
4. *[14 pts.]* Consider the integral $\iiint_E y^2 dV$, where E lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2+y^2+z^2 = z$. Sketch the region of integration and convert this integral to spherical coordinates and find the limit of integration. **DO NOT EVALUATE**.
5. *[15 pts.]* A particle is moved by the force $\vec{F}(x, y) = y^2\vec{\imath} + 2xy\vec{\jmath}$ along the curve $\vec{r}(t) = (2 + t)\vec{\imath} + t\vec{\jmath}$ from the point (2,0) to the point (3,1). What is the work done by the force?
6. *[15 pts.]* For the vector field $\vec{F}(x, y, z) = x\vec{\imath} + e^y \sin z\vec{\jmath} + e^y \cos z\vec{k}$ find its curl and divergence. If the field $\vec{F}$ is conservative, find the function f such that $\nabla f = \vec{F}$.
7. *[12 pts.]* Use Green's Theorem to evaluate $\int_C (e^{\sqrt{x}} - xy^2)dx + (5 + \cos(y^3))dy$, where C is the boundary of the region bounded by the parabolas $y = x^2$ and $x = y^2$ with positive orientation. Sketch the region bounded by the curve C and show the orientation of C.

Bonus Problem (*[10 pts]*, **no partial credit**).

Find div $\left(\dfrac{\vec{r}}{|\vec{r}|}\right)$, if $\vec{r} = < x, y, z >$.