1. A large tank initially contains 10 L of fresh water. A brine containing 20 g/L of salt flows into the tank at a rate of 3 L/min. The solution inside the tank is kept well stirred and flows out of the tank at the rate 2 L/min. Determine the concentration of salt in the tank as a function of time.

2. An object with temperature 150° is placed in a freezer whose temperature is 30°. Assume that the temperature of the freezer remains essentially constant.

   (a) If the object is cooled to 120° after 8 min, what will its temperature be after 18 min?

   (b) When will its temperature be 60°?

3. Determine (without solving the problem) an interval in which the solution to the initial value problem

$$(4-t^2)y' + 2ty = 3t^2, \qquad y(1) = -3$$

   is certain to exist.

4. Solve the initial value problem

$$y' = \frac{t^2}{1+t^3}, \qquad y(0) = y_0$$

   and determine how the interval in which the solution exists depends on the initial value y_0.

5. Solve the following initial value problem

$$\sqrt{y}dt + (1+t)dy = 0 \quad y(0) = 1.$$

6. Find the general solution to the equation

$$(t^2-1)y' + 2ty + 3 = 0$$

7. For the equation $\dfrac{dy}{dt} = y^3 - 2y^2 + y$

   (a) find the equilibrium solutions

   (b) sketch the phase line and determine whether the equilibrium solutions are stable, unstable, or semistable

   (c) graph some solutions

   (d) if $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty \le y_0 \le \infty$, find the limit of $y(t)$ when $t \to \infty$ and the limit of $y(t)$ when $t \to -\infty$

   (e) solve the equation.

8. Solve the initial value problem

$$(ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x)dx + (xe^{xy}\cos(2x) - 3)dy = 0, \qquad y(0) = -1$$

9. Find an integrating factor for the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

   and then solve the equation.

10. Solve the initial value problem

$$6y'' - 5y' + y = 0, \qquad y(0) = 4, y'(0) = 0$$

11. Find the general solution to the equation

$$4y'' - 12y' + 9y = 0$$

12. Find the interval on which the solution of the initial value problem

$$x^3 y'' + \frac{x}{\sin x} y' - \frac{2}{x-5} y = 0, \quad y(2) = 6, \quad y'(2) = 7$$

is certain to exist.

13. Given that $y_1(x) = x$ is a solution to

$$x^2 y'' + xy' - y = 0.$$

Use reduction of order to find a second solution of this equation on $(0, +\infty)$.

14. A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in. then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position u of the mass at any time t. Determine the frequency, period and amplitude of the motion.

15. A mass weighting 8 lb is attached to a spring hanging from the ceiling and comes to rest at its equilibrium position. At $t = 0$, an external force $F(t) = 2\cos 2t$ lb is applied to the system. If the spring constant is 10 lb/ft and the damping constant is 1 lb-sec/ft, find the steady-state solution for the system. What is the resonance frequency for the system?

16. Find the general solution of the equation

   (a) $y'' + 6y' + 9y = \frac{e^{-3x}}{1+2x}$
   (b) $y'' + 2y' + y = 4e^{-t}$, $y(0) = 2$, $y'(0) = 1$
   (c) $y'' + 4y = 32\sin 2t - 32t\cos 2t$
   (d) $y'' + 9y = (t^2 + t)\cos t$
   (e) $y'' - 3y' + 2y = te^t \sin t$

17. Find the Laplace transform of the given function.

   (a) $f(t) = \begin{cases} \frac{t}{2}, & 0 \le t < 6 \\ 3, & t \ge 6 \end{cases}$
   (b) $f(t) = (t^2 - 2t + 2)u_1(t)$
   (c) $f(t) = \int_0^t (t-\tau)^2 \cos 2\tau d\tau$
   (d) $f(t) = t\cos 3t$
   (e) $f(t) = e^t \delta(t-1)$

18. Find the inverse Laplace transform of the given function.

   (a) $F(s) = \dfrac{2s+6}{s^2 - 4s + 8}$
   (b) $F(s) = \dfrac{e^{-2s}}{s^2 + s - 2}$

19. Solve the initial value problem using the Laplace transform:

   (a) $y'' + 4y = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$, $y(0) = y'(0) = 0$

(b) $y'' + 2y' + 3y = \delta(t - 3\pi)$, $y(0) = y'(0) = 0$

(c) $y'' + 4y' + 4y = g(t)$, $y(0) = 2$, $y'(0) = -3$

20. Find the general solution of the system. Classify the critical point (0,0) as to type, and determine whether it is stable or unstable.

(a) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}$

(b) $\mathbf{x}' = \begin{pmatrix} -3 & -1 \\ 1 & -1 \end{pmatrix} \mathbf{x}$

(c) $\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} \mathbf{x}$

21. Find the general solution of the system using variation of parameters and Laplace Transform, if possible.

(a) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}$

(b) $\mathbf{x}' = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} \mathbf{x} + \begin{pmatrix} t^{-3} \\ -t^{-2} \end{pmatrix}$

22. For the equation $y'' + xy' + 2y = 0$

(a) Seek its power series solution about $x_0 = 0$; find the recurrence relation.

(b) Find the first four terms in each of two solutions y_1 and y_2. Show that $W[y_1, y_2](0) \neq 0$.

23. Determine a lower bound for the radius of convergence of series solution for the equation

$$(x^2 + x)y'' + 3y' - 6xy = 0$$

about

(a) $x_0 = -2$

(b) $x_0 = -\dfrac{1}{3}$

(c) $x_0 = 2$

24. Determine $y'''(1)$ if $y(x)$ is the solution of the initial value problem

$$x^2 y'' + (1 + x)y' + 3(\ln x)y = 0, \qquad y(1) = 2, y'(1) = 0$$