over sections 3.6 – 3.8, 6.1 – 6.6, 7.1 – 7.3, 7.5, 7.6

1. Find the general solution of the equation $y'' + 6y' + 9y = \dfrac{e^{-3x}}{1+2x}$

2. A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in. then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position u of the mass at any time t. Determine the frequency, period and amplitude of the motion.

3. A mass weighting 8 lb is attached to a spring hanging from the ceiling and comes to rest at its equilibrium position. At $t = 0$, an external force $F(t) = 2\cos 2t$ lb is applied to the system. If the spring constant is 10 lb/ft and the damping constant is 1 lb-sec/ft, find the steady-state solution for the system. What is the resonance frequency for the system?

4. Find the Laplace transform of the given function.

   (a) $f(t) = \begin{cases} \dfrac{t}{2}, & 0 \le t < 6 \\ 3, & t \ge 6 \end{cases}$

   (b) $f(t) = (t^2 - 2t + 2)u_1(t)$

   (c) $f(t) = \displaystyle\int_0^t (t-\tau)^2 \cos 2\tau d\tau$

   (d) $f(t) = t\cos 3t$

   (e) $f(t) = e^t \delta(t-1)$

5. Find the inverse Laplace transform of the given function.

   (a) $F(s) = \dfrac{2s+6}{s^2 - 4s + 8}$

   (b) $F(s) = \dfrac{e^{-2s}}{s^2 + s - 2}$

6. Solve the initial value problem using the Laplace transform:

   (a) $y'' + 4y = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$, $y(0) = y'(0) = 0$

   (b) $y'' + 2y' + 3y = \delta(t - 3\pi)$, $y(0) = y'(0) = 0$

   (c) $y'' + 4y' + 4y = g(t)$, $y(0) = 2$, $y'(0) = -3$

7. Find A^{-1} if $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$

8. Find BA if $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$, $B = \begin{pmatrix} i & 3 \\ 2 & -2i \end{pmatrix}$

9. Find all eigenvalues and eigenvectors of the martix

$$\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

10. Find the general solution of the system

   (a) $\begin{cases} x_1' = x_1 + x_2 \\ x_2' = 4x_1 - 2x_2 \end{cases}$

   (b) $\begin{cases} x_1' = 2x_2 - 3x_1 \\ x_2' = -x_1 - x_2 \end{cases}$