Chapter I **Introduction**
Section 1.1 **Background**

Definition Equation that contains some derivatives of an unknown function is called a *differential equation*.

Differential equations arise in a variety of subject areas, including physical sciences, economics, medicine, psychology, and operation research.

Examples:

(a) In case of free fall, an object is released from a certain height above the ground and falls under the force of gravity (we are assuming here that gravity is the *only* force acting on the object an that force is a constant). Newtons second law $\vec{F} = m\vec{a}$ can be applied to the falling object. Here m is a mass of the object, $\vec{a}$ is its acceleration, $\vec{F}$ is the total force acting on the object. This leas to the equation

$$m\frac{d^2h}{dt^2} = -mg$$

,

where h is the height above the ground, $\frac{d^2h}{dt^2} = a$, g is gravitational acceleration, and $F = -mg$ is the force due to gravity. This is the differential equation containing the second derivative of the unknown height h as a function of time.

(b) In the case of radioactive decay, we begin from the premise that the rate of decay is proportional to the amount of radioactive substance present. This leads to the equation

$$\frac{dA}{dt} = -kA \ , k > 0,$$

where $A > 0$ is the unknown amount of radioactive substance present at time t and k is a constant.

(c) A classic application of diff. eq. is found in the study of an electric circuit consisting of a resistor, an inductor, and a capacitor driven by an electromotive force. Here an application of Kirchhoff's law leads to the equation

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t),$$

where L is the inductance, R is the resistance, C is the capacitance, $E(t)$ is the electromotive force, $q(t)$ is the charge of capacitor, and t is the time.

(d) In the study of the gravitational equilibrium of a star, an applications of Newton'slaw of gravity and of the Stefan-Boltzmann law of gases leas to the equilibrium equation

$$\frac{1}{r^2}\frac{d}{dr}(\frac{r^2}{\rho}\frac{dP}{dr}) = -4\pi\rho G,$$

where P is the sum of the gas kinetic pressure and the radiation pressure, r is the distance from the center of the star, ρ is the density of matter, and G is the gravitation constant.

(e) In psychology, one model of the learning of a task involves the equation

$$\frac{dy/dt}{y^{3/2}(1-y)^{3/2}} = \frac{2p}{\sqrt{n}}.$$

Here the variable y represents the learner's skill level as a function of time t. The constants p and n depend on the individual learner and the nature of the task.

(f) The wave equation

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0,$$

where t is the time, x is the location along the string, c is a wave speed (constant), and $u = u(t, x)$ is the displacement of the string.

Definition A differential equation involving only ordinary derivatives with respect to a single variable is called an *ordinary differential equations* or ODE. A differential equation involving partial derivatives with respect to more then one variable is a *partial differential equations* or PDE.

The equations from examples (a)–(e) are the ODEs, and the equation from example (f) is the PDE.

Definition The *order* of a differential equation is the order of the highest-order derivatives present in equation.

The equations from the examples (a),(c),(d) are the second-order equations, the equations from the examples (b),(e) are the first-order equations, and the equation from the example (f) is a first-order equation with respect to t and a first-order equation with respect to x.

Definition An ODE is *linear* if it has format

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + a_1(x)\frac{dy}{dx} + a_0(x)y = F(x),$$

where $a_n(x)$, $a_{n-1}(x)$,...,$a_0(x)$ and $F(x)$ depend only on variable x. If an ODE is not linear, then we call it **nonlinear**.

For example, equations

$$\frac{d^2y}{dx} + y\frac{dy}{dx} = 2,$$

$$2\frac{d^3y}{dx^3} + y^4 = 0$$

are nonlinear. The first equation is nonlinear because of $y\frac{dy}{dx}$ term, and the second eq. is nonlinear because of y^4 term.

Equations

$$x^3\frac{d^2y}{dx^2} + xy = x^2,$$

$$\sin x \frac{dy}{dx} + y \tan x = \cos x$$

are linear.