Chapter 3. Mathematical methods and numerical methods involving first order equations.

Section 3.4 **Newtonian mechanics**

Mechanics is the study of the motion of objects and the effect of forces acting on those objects. **Newtonian** or **classical**, **mechanics** deals with the motion of **ordinary** objects–that is, objects that are large compared to an atom and slow moving compared with the speed of light. A model for Newtonian mechanics can be based on **Newton's laws of motion**:

1. When a body is subject to no resultant external force, it moves with a constant velocity.
2. When a body is subject to one or more external forces, the time rate of change of the body's momentum is equal to the vector sum of the external forces acting on it.
3. When one body interacts with a second body, the force of the first body on the second is equal in magnitude, but opposite in direction, to the force of the second body on the first.

These laws are extremely useful for studying of ordinary objects in an **inertial reference frame**–that is frame in which an undisturbed body moves with a constant velocity. We can express Newtons second law as

$$\frac{dp}{dt} = F(t, v, x),$$

where $F(t, v, x)$ is the resultant force on the body at time t, location x, and velocity v, and $p(t)$ is the momentum of the body at time t. The momentum is the product of the mass of the body and its velocity–

$$p(t) = mv(t).$$

We can express second Newton's law as

$$m\frac{dv}{dt} = ma = F(t, v, x),$$

where $a = \frac{dv}{dt}$ is the acceleration of the boy at time t.

We will focus on situations where the force F does not depend on x. We can regard the first order equation

$$m\frac{dv}{dt} = F(t, v)$$

in $v(t)$.

Procedure for Newtonian models

(a) Determine all relevant forces acting on the object being studied. It is helpful to draw a simple diagram of the object that depicts these forces.

(b) Choose an appropriate axis or coordinate system in which to represent the motion of the object and the forces acting on it.

(c) Apply Newton's second law to determine the equations of motion for the object.

You have to remember that the gravitational acceleration is approximately equal in the U.S. Customary System $g = 32$ ft/sec^2, and in the meter-kilogram-second system $g = 9.81$ m/sec^2.

Example 1. An object of mass m is given an initial downward velocity v_0 and allowed to fall under the influence of gravity. Assuming the gravitational force is constant and the force due to air resistance is proportional to the velocity of the object, determine the equation of motion for this body.

Example 2. A 400-lb object is released from rest 500 ft above the ground and allowed to fall under the influence of gravity. Assuming that the force in pounds due to air resistance is $-10v$, where v is the velocity of the object in ft/sec, determine the equation of motion of the object. When will the object hit the ground?

Example 3. An object of mass 8 kg is given an upward initial velocity of 20 m/sec and then allowed to fall under the influence of gravity. Assume that the force in newtons due to air resistance is $-16v$, where v is the velocity of the object in m/sec. Determine the equation of motion of the object. If the object is initially 100 m above the ground, determine when the object will strike the ground.

Example 4. A parachutist whose mass is 100 kg drops from a helicopter hovering 3000 m above the ground and falls under the influence of gravity. Assume that the force due to air resistance is proportional to the velocity of the parachutist, with the proportionality constant $b_1 = 20$ N-sec/m when the chute is closed and $b_2 = 100$ N-sec/m when the chute is open. If the chute does not open until 30 sec after the parachutist leaves the helicopter, after how many second will he hit the ground? If the chute does not open until 1 min after the parachutist leaves the helicopter, after how many second will he hit the ground?

Example 5. In some situations the resistive drag force on an object is proportional to a power of $|v|$ other than 1. Then when the velocity is positive, Newton's second law for the falling object generalizes to

$$m\frac{dv}{dt} = mg - bv^r,$$

where m and g have their usual interpretation an $b > 0$ and the exponent r are experimental constants. Express the solution to this equation for $r = 2$.