1.3 **Direction Fields**

Technique that is useful in graphing the solutions to a first-order differential equation is to sketch the direction field for the equation. To describe this method, we need to make a general observation. Namely, a first-order equation

$$\frac{dy}{dx} = f(x, y)$$

specifies a slope at each point in the xy-plane where f is defined.

A plot of short line segments drawn at various points in the xy-plane showing the slope of the solution curve there is called a **direction field** for the differential equation. Because the direction field gives the "flow of solutions", it facilitates the drawing of any particular solution (such as the solution to an initial value problem).

Method of Isoclines

Definition 2. An *isocline* for the differential equation

$$y' = f(x, y)$$

is a set of points in the xy-plane where all the solutions have the same slope dy/dx; thus, it is a level curve for the function $f(x, y)$.

For example, if $y' = x+y$, the isoclines are the straight lines $x+y = C$, here C is an arbitrary constant. But C can be interpreted as the numerical value of the slope dy/dx of every solution curve as it crosses the isocline. To implement the method of isoclines for sketching direction fields, we draw hash marks with slope C along the isocline $f(x, y) = C$ for a few selected values of C. If we when erase the underlying isocline curves, the hash marks constitute a pard of the direction field for the differential equation.

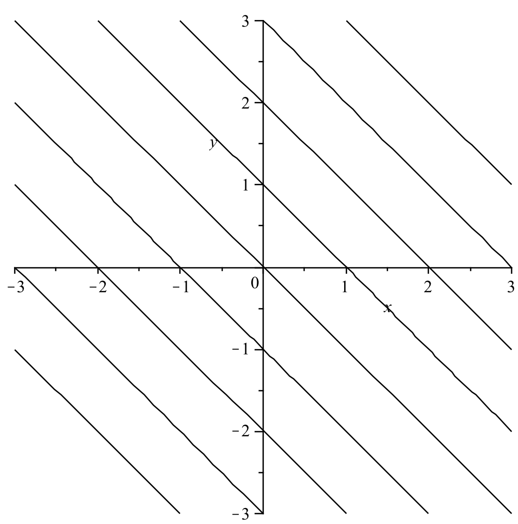

Figure 1. Isoclines for $y' = x + y$

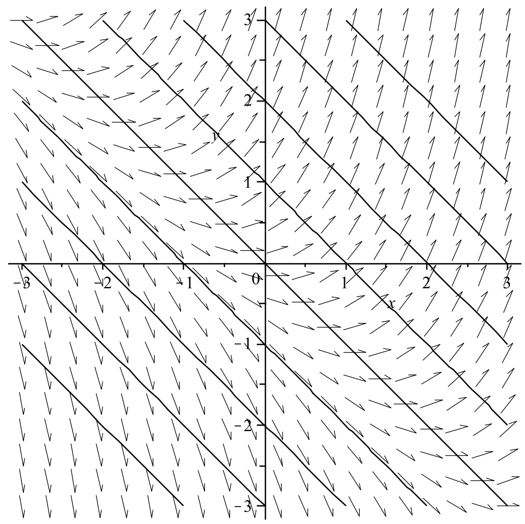

Figure 2. Direction field for $y' = x + y$

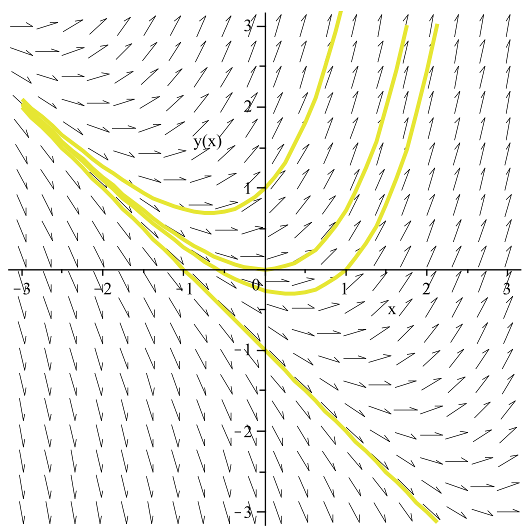

Figure 3. Solutions to $y' = x + y$

Remark. When the isoclines curves are complicated, this method is not practical.