

Find the general solution to the following systems.

$$1. \begin{cases} \frac{dx}{dt} = 3x - y + z \\ \frac{dy}{dt} = x + y + z \\ \frac{dz}{dt} = 4x - y + 4z \end{cases}$$

(Answer: $x(t) = C_1e^t + C_2e^{2t} + C_3e^{5t}$, $y(t) = C_1e^t - 2C_2e^{2t} + C_3e^{5t}$, $z(t) = -C_1e^t - 3C_2e^{2t} + 3C_3e^{5t}$.)

$$2. \begin{cases} \frac{dx}{dt} = x - 2y - z \\ \frac{dy}{dt} = y - x + z \\ \frac{dz}{dt} = x - z \end{cases}$$

(Answer: $x(t) = C_1 + 3C_2e^{2t}$, $y(t) = -2C_2e^{2t} + C_3e^{-t}$, $z(t) = C_1 + C_2e^{2t} - 2C_3e^{-t}$.)

$$3. \begin{cases} \frac{dx}{dt} = 3x - 2y - z \\ \frac{dy}{dt} = 3x - 4y - 3z \\ \frac{dz}{dt} = 2x - 4y \end{cases}$$

(Answer: $x(t) = C_1e^{2t} + C_3e^{-5t}$, $y(t) = C_2e^{2t} + 3C_3e^{-5t}$, $z(t) = (C_1 - 2C_2)e^{2t} + 2C_3e^{-5t}$.)

$$4. \begin{cases} \frac{dx}{dt} = 2x - y - z \\ \frac{dy}{dt} = 3x - 2y - 3z \\ \frac{dz}{dt} = 2z - x + y \end{cases}$$

(Answer: $x(t) = C_1 + C_2e^t$, $y(t) = 3C_1 + C_3e^t$, $z(t) = -C_1 + (C_2 - C_3)e^t$.)