MATH 308 **Fall 2007** **Practice Exam I**

1. For each of the differential equations below state its order and whether it is linear or nonlinear.

	Equation	order	linear/nonlinear
(i)	$y'' + ty' + t^2 y = 0$		
(ii)	$y''' + 2y' - 3y^4 = 0$		
(iii)	$y' = y^2 + 4y - 5$		
(iv)	$e^{2t}y'' - 2y' + 3y = \sqrt[3]{t}$		
(v)	$(y')^2 + 2y = \ln t$		

2. Given the differential equation $y' - 2y = e^{2t}$ with the initial condition $y(0) = 2$. Which of the following will is the correct solution to this problem?

(a) $y(t) = e^{2t} + e^{-2t}$
(b) $y(t) = (t + 2)e^{2t}$
(c) $y(t) = (t + 2)e^{-2t}$
(d) $y(t) = 2e^{2t} - e^{-2t}$

3. Which of the following will be an integrating factor for the differential equation:

$$ty' - 2y = 2\cos 2t?$$

(a) $\frac{1}{t^2}$
(b) e^{-2t}
(c) $-t^2$
(d) $-2t$

4. The Existence and Uniqueness Theorem guarantees that the solution to

$$t^3\, y'' + \frac{t}{\sin t}\, y' - \frac{2}{t-5}\, y = 0, \quad y(2) = 6, \quad y'(2) = 7$$

uniquely exists on

(a) $(-\pi, \pi)$
(b) $(0, \pi)$
(c) $(5, \infty)$
(d) $(0, 5)$

5. All of the following pairs of functions form a fundamental set of solutions to some second order differential equation on $(-\infty, \infty)$ EXCEPT

(a) $1, \quad e^{-t}$
(b) $\cos t, \quad \sin(t + 2\pi)$
(c) $e^{-2t}\cos 2t, \quad e^{-2t}\sin 2t$
(d) $e^{5t}, \quad e^{5t-1}$

6. Which of the following will be a particular solution to the equation

$$4y'' + 4y' + y = 24x\mathrm{e}^{\frac{x}{2}}?$$

(a) $x^2(Ax+B)\mathrm{e}^{\frac{x}{2}}$
(b) $(Ax+B)\mathrm{e}^{\frac{x}{2}}$
(c) $x(Ax+B)\mathrm{e}^{\frac{x}{2}}$
(a) $(Ax+B)\sin\frac{x}{2} + (Cx+D)\cos\frac{x}{2}$

7. All the following differential operators are linear EXCEPT
(a) $L[y] = y'' - 3y' + y^3$
(b) $L[y] = y'' + y' + 2y$
(c) $L[y] = y'' + \sin xy' + \cos xy$
(d) $L[y] = y'' + xy' + (x-1)y$

8. Find a **general** solution to the equation

$$y'' + 6y' + 9y = \frac{e^{-3x}}{1+2x}$$

9. Solve the following initial value problem:

$$y' = \frac{4x-3}{2y+6}, \quad y(1) = -5.$$

Write your solution in the **explicit** form.

10. Find a **particular** solution to the equation

$$4y'' + y' = 4x^3 + 48x^2 + 1$$

11. Find a Wronskian of two solutions of

$$xy'' - (x+1)y' - y = 0; \quad x > 0$$

provided $W[y_1, y_2](1) = 1$.

12. Solve the following initial value problem

$$4y'' + 12y' + 13y = 0, \;\; y(0) = 1, \;\; y'(0) = -2$$

13. Find the general solution to

$$\frac{1}{x}\frac{dy}{dx} - \frac{2y}{x^2} = x\cos x.$$

14. Given that $y_1(x) = x$ is a solution to

$$x^2y'' + xy' - y = 0,$$

find the general solution to this equation on $(0, +\infty)$.