MATH 308 Fall 2007 Practice Exam II

1. A 2-kg mass is attached to a spring with stiffness $k = 50$ N/m. The mass is displaced 1/4 m to the left of the equilibrium point and given a velocity of 1 m/sec to the left. Neglecting damping,

a) Set up an initial value problem for this system.
b) Find the equation of motion of the mass.
c) Find the amplitude, period and frequency of the motion.

2. A tank initially contains 100 L of fresh water. A brine containing 200 g/L of salt salt flows into the tank at rate of 3 L/min. The solution inside the tank is kept well stirred and flows out of the tank at the rate 2 L/min. How long does it take for the concentration of salt in tank to become 500 g?

3. A 400-lb object is released from rest 500 ft above the ground and allowed to fall under the influence of gravity. Assume that the force in pounds due to air resistance is $-10v$, where v is the velocity of the object in ft/sec.

a) Determine the equation of motion of the object.
b) When will the object hit the ground?

4. An object with temperature 150^0 is placed in a freezer whose temperature is 30^0. Assume that the temperature of the freezer remains essentially constant.

a) If the object is cooled to 120^0 after 8 min, what will its temperature after 18 min?
b) When will its temperature be 60^0?

5. Find the Laplace transform of $f(t) = \frac{t}{3}$ using the definition of the Laplace transform.

6. Find $\mathcal{L}^{-1}\left\{\dfrac{-s-7}{(s+1)(s-2)}\right\}$.

7. Find $\mathcal{L}\{2\cos 3t - 4\mathrm{e}^t t^3\}$.

8. Solve the initial value problem using the method of Laplace transform

$$y'' + y = t, \quad y(\pi) = 0, \quad y'(\pi) = 1.$$

9. Find $\mathcal{L}\{\mathrm{e}^t(u(t-2) + 2t)\}$.

10. Find the Laplace transform for the function

$$f(t) = \begin{cases} \sin t, & 0 \le t < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le t < \pi \\ \mathrm{e}^t, & t \ge \pi \end{cases}$$

11. Find $\mathcal{L}^{-1}\left\{\dfrac{\mathrm{e}^{-3s}}{s^2+9}\right\}$.