MATH 308 Fall 2007 Practice Before Final

1. Solve the IVP

$$(x^2+1)\frac{dy}{dx} + xy = x, \quad y(0) = 1.$$

2. Suppose the object takes 40 min to cool from 30^0C to 24^0C in a room that is kept at 20^0C.

(a) What was the temperature of the object 15 min after it was 30^0?

(b) How long will it take the object to cool down to 21^0C?

3. A 50-kg object is released from rest 100 m above the ground and allowed to fall under the influence of gravity. Assume that the force in Newtons due to air resistance is $-5v$, where v is the velocity of the object in m/sec.

a) Determine the equation of motion of the object.

b) When will the object hit the ground?

4. Given that $y_1(x) = x^{-1}$ is a solution to

$$x^2y'' - 2xy' - 4y = 0,$$

find the general solution to this equation on $(0, +\infty)$.

5. Solve the following IVP

$$y'' - 2y' - 3y = 3x^2 - 5, \quad y(0) = 0, \;\; y'(0) = 1.$$

6. Find the general solution to the equation

$$y'' - 2y' + y = \frac{\mathrm{e}^x}{x}$$

using the method of variation of parameters.

7. A 2-kg mass is attached to a spring hanging from the ceiling, thereby causing the spring to stretch 20 cm upon coming to rest at equilibrium. At $t = 0$, the mass is displaced 5 cm below the equilibrium position and released. At this same instant, an external force $F(t) = 0.3\cos t$N is applied to the system. If the dumping constant for the system is 5 N-sec/m, determine the equation of motion for the mass.

8. Two large tanks each holding 50 L of liquid, are interconnected by pipes, with the liquid flowing from tank A into tank B at a rate of 5 L/min and from B to A at a rate of 3 L/min. The liquid inside each tank is kept well stirred. A brine solution with a concentration of 0.4 kg/L of salt flows into tank A at a rate of 6 L/min. The solution flows out of the system, from tank A at 4 L/min and from tank B at 2 L/min. Initially, tank A contains 5 kg of salt and tank B contains pure water. Determine the mass of salt at each tank at time $t > 0$.

9. Use the elimination method to find the general solution to the system

$$\begin{cases} \dfrac{dx}{dt} = 2x - 4y \\ \dfrac{dy}{dt} = x - 3y + 3\mathrm{e}^{-t} \end{cases}$$

10. Find $\mathcal{L}\{2t^2\mathrm{e}^{-t} - t + \cos 4t\}$.

11. Find $\mathcal{L}^{-1}\left\{\dfrac{7s^2+23s+30}{(s-2)(s^2+2s+5)}\right\}$.

12. Solve the IVP using the method of Laplace transform

$$y''-4y=4t-8\mathrm{e}^{-2t}, \quad y(0)=0, \;\; y'(0)=5.$$

13. Find $\mathcal{L}^{-1}\left\{\dfrac{\mathrm{e}^{-3s}(s-5)}{(s+1)(s+2)}\right\}$.

14. Find the Laplace transform for the function

$$f(t)=\begin{cases}\sin 2t, & 0\le t<\frac{\pi}{2}\\ t, & \frac{\pi}{2}\le t<2\\ \mathrm{e}^{-t}, & t\ge 2\end{cases}$$

15. Find the general solution to the system

$$\begin{cases}\dfrac{dx}{dt}=4x-y-z\\ \dfrac{dy}{dt}=x+2y-z\\ \dfrac{dz}{dt}=x-y+2z\end{cases}$$