MATH 308 Sheet 2

We have to iterate the algorithm that realizes an Euler's method to construct approximations to the solution of the initial value problem for first-order differential equation:

$$\frac{dy}{dx} = f(x,y), \qquad y(x_0) = y_0.$$

As you know, an Euler's method can be summarized by the recursive formulas

$$x_{n+1} := x_0 + (n+1)h,$$

$$y_{n+1} := y_n + f(x_n, y_n)(x - x_n), \quad n = 0, 1, 2, \ldots$$

Lets construct numerical approximations to the initial value problem

$$\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x} - y^2, \quad y(1) = 1$$

on the interval $1 < x < 2$.

To iterate this algorithm, we use the **for** k **from** *start* **to** *finish* **do..od** construction. The following sequence of Maple commands performs ten iterations of this procedure with step $h = 0.1$ and therefore computes approximate values at the points $x = 1, 1.1, 1.2, ...1.9, 2.0$. These values are contained in a sequence of points named *eseq*.

```
> f:=(x,y) -> 1/x^2-y/x-y^2;inits:=y(1)=1;
```

$$f := (x,y) \to \frac{1}{x^2} - \frac{y}{x} - y^2$$

$$inits := y(1) = 1$$

Notice, that the function above is an arrow-defined function an not a Maple expression.

```
> x:=1:y:=1:h:=0.1:  # initialize x and y an the step size
> eseq:=[x,y];  # input the initial conditions into eseq
```

$$eseq := [1, 1]$$

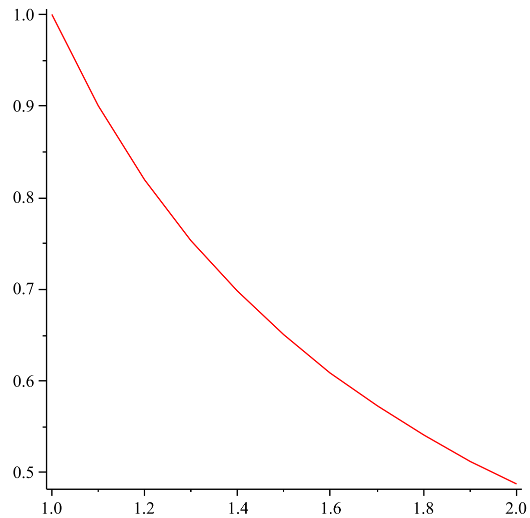
```
> for i from 1 to 10 do
  y:=evalf(y+h*f(x,y)):  # compute the new value of y
  x:=x+h:  # update the new value of x
  eseq:=eseq,[x,y]:  # ad the new point
  od:
> x:='x':y:='y':h:='h':
```

To display the contest of the solution values in *eseq*, type the variable name.

```
> eseq;
```

[1, 1], [1.1, .9], [1.2, .8198264463], [1.3, .7537404800], [1.4, .6981195696], [1.5, .6505372009], [1.6, .6092926336], [1.7, .5731505927], [1.8, .5411877679], [1.9, .5126975583], [2.0, .4871284287]

To plot the approximation between $x = 1$ and $x = 2$, we could use `plot([eseq]);`.



1. Use Euler's method to find approximations to the solution of the initial value problem

$$y' = 1 - \sin y, \quad y(0) = 0,$$

taking 10 steps.

2. Use Euler's method to find approximations to the solution of the initial value problem

$$y' = 1 + x^2, \quad y(0) = 0,$$

taking 10 steps.