# MATH 308-519 Spring 2010 Practice Test I

1. A large tank initially contains 10 L of fresh water. A brine containing 20 g/L of salt flows into the tank at a rate of 3 L/min. The solution inside the tank is kept well stirred and flows out of the tank at the rate 2 L/min. Determine the concentration of salt in the tank as a function of time.

2. Suppose that a sum $S_0$ is invested at an annual rate of return $r$ compounded continuously.

   (a) Find the time $T$ required for the original sum to double in value as a function of $r$.

   (b) Determine $T$ if $r = 7\%$.

   (c) Find the return rate that must be achieved if the initial investment is to double in 8 years.

3. An object with temperature $150^0$ is placed in a freezer whose temperature is $30^0$. Assume that the temperature of the freezer remains essentially constant.

   (a) If the object is cooled to $120^0$ after 8 min, what will its temperature be after 18 min?

   (b) When will its temperature be $60^0$?

4. Determine (without solving the problem) an interval in which the solution to the initial value problem

$$(4 - t^2)y' + 2ty = 3t^2, \quad y(1) = -3$$

   is certain to exist.

5. Solve the initial value problem

$$y' = \frac{t^2}{1 + t^3}, \quad y(0) = y_0$$

   and determine how the interval in which the solution exists depends on the initial value $y_0$.

6. Solve the following initial value problem

$$\sqrt{y}dt + (1 + t)dy = 0 \quad y(0) = 1.$$

7. Find the general solution to the equation

$$(t^2 - 1)y' + 2ty + 3 = 0$$

8. Solve the initial value problem

$$(ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x)dx + (xe^{xy}\cos(2x) - 3)dy = 0, \quad y(0) = -1$$