MATH 308-519 Spring 2010 Practice Test II

1. Solve the following systems:

   (a) $\begin{cases} 2x_1 - 3x_2 = 4 \\ x_1 + 2x_2 = -5 \end{cases}$

   (b) $\begin{cases} x_1 - 3x_2 = 5 \\ -2x_1 + 6x_2 = 1 \end{cases}$

   (c) $\begin{cases} -2x_1 + x_2 = 4 \\ 6x_1 - 3x_2 = -12 \end{cases}$

2. Find the general solution of the system of equations/solve the initial value problem:

   (a) $\vec{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \vec{x}, \qquad \vec{x}(0) = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

   (b) $\vec{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \vec{x}$

   (c) $\vec{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \vec{x}, \qquad \vec{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

3. Use variation of parameters to find the general solution for the system

$$\vec{x}' = \begin{pmatrix} -4 & -2 \\ 6 & 3 \end{pmatrix} \vec{x} + \begin{pmatrix} \dfrac{2}{e^t - 1} \\ -\dfrac{3}{e^t - 1} \end{pmatrix}$$

4. A mass of 20 g stretches a spring 5 cm. Suppose that the mass is also attached to a viscous damper with a damping constant of 400 dyne-s/cm. The mass is pulled down an additional 2 cm and then released. Formulate the initial value problem that governs the motion of the mass.

5. Determine which of the following operators are linear:

   (a) $L_1[y] = y'' + e^y y$

   (b) $L_2[y] = ty'' + t^2 y' + (t^3 - 1)y$

   (c) $L_3[y] = t(y'')^2 + y$

   (d) $L_4[y] = 2y'' + 3y' + 4y$

   (e) $L_5[y] = 2y'' + 2(y')^2 + 4y$

6. Determine the longest interval in which the solution to the initial value problem

$$(x-3)y'' + xy' + y \ln x = 0 \qquad y(1) = 0 \quad y'(1) = 1$$

   is certain to have a unique twice differentiable solution.

7. If $W[f, g] = t^2 e^t$ and $f(t) = t$, find $g(t)$.

8. Verify that the functions $y_1(x) = x$, $y_2(x) = \sin x$ are solutions of the differential equation

$$(1 - x \cot x)y'' - xy' + y = 0, \qquad 0 \le x \le \pi.$$

   Do they constitute a fundamental set of solutions?