

MATH 308-519 Spring 2011 Practice Test III

1. Find the general solution of the equation:

(a) $y'' - 3y' + 2y = x \cos x$

(b) $y'' - 9y' = e^{-3x}(x^2 + \sin 3x)$

(c) $y'' - 2y' + y = \frac{e^x}{x}$

2. Solve the Cauchy-Euler equation

$$x^2y'' - 4xy' + 6y = 0$$

3. Given that $y_1(x) = x^{-2}$ is a solution to

$$x^2y'' + 6xy' + 6y = 0,$$

Find the second linearly independent solution to this equation.

4. Find $\mathcal{L}\{f(t)\}(s)$ if

$$f(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 3 - t, & 1 < t \leq 2 \\ 1, & t > 2 \end{cases}$$

5. Find $\mathcal{L}\{t \cos t + e^{3t} \sin 2t + t^5 e^{2t}\}$

6. Find $\mathcal{L}^{-1}\left\{\frac{3s+2}{(s^2-4)(s+1)}\right\}$

7. Solve the initial value problem using the method of Laplace transform

$$y'' + 2y' + y = 4e^{-t}, \quad y(0) = 2, \quad y'(0) = -1$$