

MATH 308-518,519,520 Spring 2013 Practice Test I

over sections 1.1–1.3, 2.1–2.6, 3.1

1. A large tank initially contains 10 L of fresh water. A brine containing 20 g/L of salt flows into the tank at a rate of 3 L/min. The solution inside the tank is kept well stirred and flows out of the tank at the rate 2 L/min. Determine the concentration of salt in the tank as a function of time.
2. Suppose that a sum S_0 is invested at an annual rate of return r compounded continuously.
 - (a) Find the time T required for the original sum to double in value as a function of r .
 - (b) Determine T if $r = 7\%$.
 - (c) Find the return rate that must be achieved if the initial investment is to double in 8 years.
3. An object with temperature 150° is placed in a freezer whose temperature is 30° . Assume that the temperature of the freezer remains essentially constant.
 - (a) If the object is cooled to 120° after 8 min, what will its temperature be after 18 min?
 - (b) When will its temperature be 60° ?
4. Determine (without solving the problem) an interval in which the solution to the initial value problem

$$(4 - t^2)y' + 2ty = 3t^2, \quad y(1) = -3$$

is certain to exist.

5. Solve the initial value problem

$$y' = \frac{t^2}{1 + t^3}, \quad y(0) = y_0$$

and determine how the interval in which the solution exists depends on the initial value y_0 .

6. Solve the following initial value problem

$$\sqrt{y}dt + (1 + t)dy = 0 \quad y(0) = 1.$$

7. Find the general solution to the equation

$$(t^2 - 1)y' + 2ty + 3 = 0$$

8. Solve the initial value problem

$$(ye^{xy} \cos(2x) - 2e^{xy} \sin(2x) + 2x)dx + (xe^{xy} \cos(2x) - 3)dy = 0, \quad y(0) = -1$$

9. Find an integrating factor for the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

and then solve the equation.

10. Solve the initial value problem

$$6y'' - 5y' + y = 0, \quad y(0) = 4, y'(0) = 0$$

11. Find the general solution to the equation

$$4y'' - 12y' + 9y = 0$$