MATH 308 – 518, 519, 520 **Homework 3** **Spring 2014**

Due Thursday, Feb. 6 at the beginning of class.

1. Determine (without solving the problem) an interval in which the solution of the given IVP is certain to exist:
$$(4 - t^2)y' + 2ty = 3t^2$$
   (a) $y(-3) = 1$
   (b) $y(0) = 2$
   (c) $y(3) = 0$

2. For the following equations state where in the ty-plane the hypotheses of Theorem 2 (section 2.4) are satisfied.
   (a) $y' = (1 - t^2 - y^2)$
   (b) $y' = \dfrac{y \cot t}{1 + y}$

3. Given the differential equation
$$\frac{dy}{dt} = 7y - y^2 - 10$$
   (a) Find the equilibrium solutions.
   (b) Sketch the phase line and determine whether the equilibrium solutions are stable, unstable, or semistable.
   (c) Using Matlab, sketch the direction field of the equation and graph of some solutions. Make sure you include the graphs of all the equilibrium solutions.

4. Show that the equation is exact and then solve it.
   (a) $(1 + e^x y + xe^x y)dx + (xe^x + 2)dy = 0$
   (b) $\dfrac{dy}{dx} = -\dfrac{2xy^2 + 1}{2x^2 y}$
   (c) $(2xy - \sec^2 x)dx + (x^2 + 2y)dy = 0$

5. Given the equation
$$(y^2 + x^2 y)dx + (x^2 y - x^3)dy = 0$$
   (a) Show that the equation is not exact.
   (b) Show that by multiplying the equation by $\mu(x, y) = \dfrac{1}{x^2 y^2}$, the resulting equation is exact. Solve the resulting exact equation.