

Due Thursday, Feb. 27 at the beginning of class.

1. Determine the longest interval in which the given initial value problem is certain to have a unique solution. Do not solve the problem.

(a)  $(1 + t^2)y'' + ty' - y = \tan t$ ,  $y(1) = y_0$ ,  $y'(1) = y_1$ .

(b)  $t(t - 3)y'' + 2ty' - y = t^2$ ,  $y(1) = y_0$ ,  $y'(1) = y_1$ .

(c)  $e^t y'' + \frac{y'}{t-3} + y = \ln t$ ,  $y(1) = y_0$ ,  $y'(1) = y_1$ .

2. Find the Wronskian for the given pair of functions.

(a)  $y_1(t) = e^{3t}$ ,  $y_2(t) = e^{-4t}$ .

(b)  $y_1(t) = e^{-t} \cos(2t)$ ,  $y_2(t) = e^{-t} \sin(2t)$ .

3. If the Wronskian  $W$  of  $f$  and  $g$  is  $3e^{4t}$ , and  $f(t) = e^{2t}$ , find  $g(t)$ .

4. Find the general solution to the given differential equation.

(a)  $y'' + 2y' - 8y = 0$

(b)  $y'' + y' + 1.25y = 0$

5. Given that  $y_1(t) = t^{-1}$  is a solution of the equation

$$t^2 y'' + 3ty' + y = 0, \quad t > 0.$$

Find a second solution of the equation.

6. Given that  $y_1(t) = \sin(t^2)$  is a solution of the equation

$$ty'' - y' + 4t^3 y = 0, \quad t > 0.$$

Find a second solution of the equation.