

Due Thursday, April, 10 at the beginning of class.

If you use convolutions, please write your answer in terms of convolution integrals.

1. Find the Laplace transform of the given function

$$(a) f(t) = \begin{cases} 0, & 0 \leq t < 1, \\ 6t - 5, & 1 \leq t < 3, \\ t^2, & t \geq 3. \end{cases}$$

$$(b) f(t) = \int_0^t e^{-(t-\tau)} \sin \tau \, d\tau$$

2. Find the inverse Laplace transform of

$$(a) \frac{e^{-2s} - 3e^{-7s}}{(s+5)^2}$$

$$(b) \frac{(s-2)e^{-s}}{s^2 - 4s + 3}$$

$$(c) \frac{1}{(s+1)^2(s^2+4)}$$

3. Solve the initial value problem using the method of Laplace transform.

$$(a) y'' + y = g(t), \quad y(0) = 0, \quad y'(0) = 1, \quad g(t) = \begin{cases} t/2, & 0 \leq t < 6, \\ 3, & t \geq 6. \end{cases}$$

$$(b) y'' + y' + \frac{5}{4}y = t - u_{\pi/2}(t) \left(t - \frac{\pi}{2}\right), \quad y(0) = y'(0) = 0$$

$$(c) y'' + 4y = \delta(t - \pi) - \delta(t - 2\pi), \quad y(0) = y'(0) = 0$$

4. Express the solution of the initial value problem

$$4y'' + 4y' + 17y = g(t), \quad y(0) = y'(0) = 0$$

in terms of a convolution integral.