

1. A large tank initially contains 10 L of fresh water. A brine containing 20 g/L of salt flows into the tank at a rate of 3 L/min. The solution inside the tank is kept well stirred and flows out of the tank at the rate 2 L/min. Determine the concentration of salt in the tank as a function of time.
2. Suppose that a sum  $S_0$  is invested at an annual rate of return  $r$  compounded continuously.
  - (a) Find the time  $T$  required for the original sum to double in value as a function of  $r$ .
  - (b) Determine  $T$  if  $r = 7\%$ .
  - (c) Find the return rate that must be achieved if the initial investment is to double in 8 years.
3. An object with temperature  $150^\circ$  is placed in a freezer whose temperature is  $30^\circ$ . Assume that the temperature of the freezer remains essentially constant.
  - (a) If the object is cooled to  $120^\circ$  after 8 min, what will its temperature be after 18 min?
  - (b) When will its temperature be  $60^\circ$ ?

4. Determine (without solving the problem) an interval in which the solution to the initial value problem

$$(4 - t^2)y' + 2ty = 3t^2, \quad y(1) = -3$$

is certain to exist.

5. Solve the initial value problem

$$y' = \frac{t^2}{1 + t^3}, \quad y(0) = y_0$$

and determine how the interval in which the solution exists depends on the initial value  $y_0$ .

6. Solve the following initial value problem

$$\sqrt{y}dt + (1 + t)dy = 0 \quad y(0) = 1.$$

7. Find the general solution to the equation

$$(t^2 - 1)y' + 2ty + 3 = 0$$

8. Given the differential equation

$$\frac{dy}{dt} = 7y - y^2 - 10$$

- (a) Find the equilibrium solutions.

(b) Sketch the phase line and determine whether the equilibrium solutions are stable, unstable, or semistable.

(c) Solve the equation.

9. Solve the initial value problem

$$(ye^{xy} \cos(2x) - 2e^{xy} \sin(2x) + 2x)dx + (xe^{xy} \cos(2x) - 3)dy = 0, \quad y(0) = -1$$

10. Find an integrating factor for the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

and then solve the equation.

11. Solve the initial value problem

$$6y'' - 5y' + y = 0, \quad y(0) = 4, y'(0) = 0$$

12. Find the general solution to the equation

$$4y'' - 12y' + 9y = 0$$

13. Find the interval(s) on which the solution of the initial value problem

$$x^3y'' + \frac{x}{\sin x}y' - \frac{2}{x-5}y = 0, \quad y(2) = 6, \quad y'(2) = 7$$

is certain to exist.

14. Find the Wronskian of two functions  $y_1(x) = x + 2x^2$  and  $y_2(x) = 2^x$ .

15. Which of the following pairs of functions form a fundamental set of solutions to some second order differential equation on  $(-\infty, \infty)$ ?

(a)  $1, e^{-t}$

(b)  $\cos t, \sin(t + 2\pi)$

(c)  $e^{-2t} \cos 2t, e^{-2t} \sin 2t$

(d)  $e^{5t}, e^{5t-1}$