MATH 308 – 530, 531 **Practice Problems for Test 2** **Spring 2016**

over sections 3.3 – 3.8, 6.1

1. $e^{2+\frac{3\pi}{4}i} =$

2. A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in. then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position u of the mass at any time t. Determine the frequency, period and amplitude of the motion.

3. A mass weighting 8 lb is attached to a spring hanging from the ceiling and comes to rest at its equilibrium position. At $t = 0$, an external force $F(t) = 2\cos 2t$ lb is applied to the system. If the spring constant is 10 lb/ft and the damping constant is 1 lb-sec/ft, find the steady-state solution for the system. What is the resonance force for the system?

4. Find the general solution of the equation

   (a) $y'' - 2y' + 5y = 0$

   (b) $y'' + 6y' + 9y = \frac{e^{-3x}}{1+2x}$

   (c) $4y'' + y' = 4x^3 + 48x^2 + 1$

5. Given that $y_1(x) = x$ is a solution to

$$x^2y'' + xy' - y = 0,$$

   find a second solution of this equation on $(0, +\infty)$.

6. Find the Laplace transform of the given function using the definition of the Laplace transform.

   (a) $f(x) = te^{3t}$.

   (b) $f(t) = \begin{cases} e^{5t} & 0 \le t < 6 \\ 3 & t \ge 6. \end{cases}$

7. Find the Laplace transform of

   (a) $f(t) = t\cos 3t$

   (b) $f(t) = t^2e^{-2t}$

8. Find the inverse Laplace transform of the given function.

   (a) $F(s) = \dfrac{2s+6}{s^2 - 4s + 8}$

   (b) $F(s) = \dfrac{e^{-2s}}{s^2 + s - 2}$