MATH 308 – 530, 531 **Practice Problems for Test 3** **Spring 2016**

over sections 6.2 – 6.6, 7.1 – 7.6, 7.8

1. Find the Laplace transform of the given function.

   (a) $f(t) = \begin{cases} \dfrac{t}{2}, & 0 \le t < 6 \\ 3, & t \ge 6 \end{cases}$

   (b) $f(t) = (t^2 - 2t + 2)u_1(t)$

   (c) $f(t) = \displaystyle\int_0^t (t-\tau)^2 \cos 2\tau d\tau$

   (d) $f(t) = e^t \delta(t-1)$

2. Find the inverse Laplace transform of the given function.

   (a) $F(s) = \dfrac{2s+6}{s^4(s^2-4s+8)}$

   (b) $F(s) = \dfrac{e^{-2s}}{s^2+s-2}$

3. Solve the initial value problem using the Laplace transform:

   (a) $y'' + 4y = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$, $y(0) = y'(0) = 0$

   (b) $y'' + 2y' + 3y = \delta(t - 3\pi)$, $y(0) = y'(0) = 0$

   (c) $y'' + 4y' + 4y = g(t)$, $y(0) = 2$, $y'(0) = -3$

4. Find A^{-1} if $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$

5. Find BA if $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$, $B = \begin{pmatrix} i & 3 \\ 2 & -2i \end{pmatrix}$

6. Find the general solution of the system

   (a) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}$

   (b) $\mathbf{x}' = \begin{pmatrix} -3 & -1 \\ 1 & -1 \end{pmatrix} \mathbf{x}$

   (c) $\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} \mathbf{x}$