

Due Thursday, April 6 at the beginning of class.

If you use convolutions, please write your answer in terms of convolution integrals.

1. Find the Laplace transform of the given function

$$(a) f(t) = \begin{cases} 0, & 0 \leq t < 1, \\ 6t - 5, & 1 \leq t < 3, \\ t^2, & t \geq 3. \end{cases}$$

$$(b) f(t) = \int_0^t e^{-(t-\tau)} \sin \tau \, d\tau$$

2. Find the inverse Laplace transform of

$$(a) \frac{e^{-2s} - 3e^{-7s}}{(s+5)^2}$$

$$(b) \frac{(s-2)e^{-s}}{s^2 - 4s + 3}$$

$$(c) \frac{1}{(s+1)^2(s^2+4)}$$

3. Solve the initial value problem using the method of Laplace transform.

$$(a) y'' + y = g(t), \quad y(0) = 0, \quad y'(0) = 1, \quad g(t) = \begin{cases} t/2, & 0 \leq t < 6, \\ 3, & t \geq 6. \end{cases}$$

$$(b) y'' + y' + \frac{5}{4}y = t - u_{\pi/2}(t) \left(t - \frac{\pi}{2}\right), \quad y(0) = y'(0) = 0$$

$$(c) y'' + 4y = \delta(t - \pi) - \delta(t - 2\pi), \quad y(0) = y'(0) = 0$$

4. Express the solution of the initial value problem

$$4y'' + 4y' + 17y = g(t), \quad y(0) = y'(0) = 0$$

in terms of a convolution integral.