4. A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in. then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position u of the mass at any time t. Determine the frequency, period and amplitude of the motion.

$mu'' + bu' + ku = 0$ (no external forces)

mass, damping const, stiffness

$b = 0$ (no damping)

$W = 3$ lb

$W = mg$

$m = \frac{W}{g} = \frac{3}{32}$

Hook's law $W = k|x|$ elongation

$3 = k\left(\frac{3}{12}\right)$ ft

$k = 12$

IVP:

$\frac{3}{32}u'' + 12u = 0$

$u(0) = -\frac{1}{12}$

$u'(0) = 2$

$u'' + 128u = 0$

$u(0) = -\frac{1}{12}$

$u'(0) = 2$

u(t)=C 1 cos(8*sqrt(2)t)+C 2 sin(8*sqrt(2)t), C 1 =-1/12, C 2 =sqrt(2)/8

A=sqrt(11/2)/12

frequency=4sqrt(2)/pi

period=pi/(4sqrt(2))