Section 2.1 **Linear Equations.**

Definition. A **linear first-order equation** is an equation that can be expressed in the form

$$a_1(x)\frac{dy}{dx} + a_0(x)y = b(x),$$

where $a_0(x)$, $a_1(x)$, $b(x)$ depend only on x.

We will assume that $a_0(x)$, $a_1(x)$, $b(x)$ are continuous functions of x on an interval I.

For now, we are interested in those linear equations for which $a_1(x)$ is never zero on I. In that case we can rewrite the equation in the **standard form**

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where $P(x) = a_0(x)/a_1(x)$ and $Q(x) = b(x)/a_1(x)$ are continuous on I.

To solve the first order linear equation:

1. Write the equation $a_1(x)\frac{dy}{dx} + a_0(x)y = b(x)$ in the standart form $\frac{dy}{dx} + P(x)y = Q(x)$.
2. Find the integrating factor $\mu(x)$ solving differential equation
$$\frac{d\mu}{dx} - P(x)\mu = 0.$$
3. Integrate the equation
$$\frac{d}{dx}[\mu y] = \mu Q(x)$$
and solve for y by dividing by $\mu(x)$.

The solution
$$y(x) = \frac{1}{\mu(x)}\left[\int \mu(x)Q(x)dx + C\right]$$
where C is a constant, to the equation is called the **general solution**.

Example 1. Find the general solution to the following equations

1. $xy' - y = -\ln x$

2. $y' - 2y = t^2 e^{2t}$

Example 2. Solve the initial value problem

1. $dy - ydx - 2x\mathrm{e}^x dx = 0, \quad y(0) = \mathrm{e} - 2$

2. $ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}, t > 0$