Chapter 2. First Order Differential Equations

Section 2.2 **Separable Equations**

$$\frac{dy}{dx} = f(x, y)$$

Sometimes a function $f(x, y)$ can be represented as a product of two functions, one of which depens ONLY on x, another depens ONLY on y, or $f(x, y) = g(x)h(y)$. Then

$$\frac{dy}{dx} = g(x)h(y).$$

Definition. A differential equation $y' = f(x, y)$ is called **separable** if it can be written in the form

$$M(x)dx + N(y)dy = 0$$

Example 1. Determine whether the given equation is separable.

1. $(t - 2y)^2 y' = 2$

2. $y^4 e^y + (t^3 + 1)y' = y'(t^3 + 1)e^{2y}$

3. $yx \ln x dx - \sqrt{y} dy + x \ln x dx = 0$

4. $y' = \cot^2\left(\frac{x}{2} + y - 1\right) + \frac{1}{2}$

How to solve a separable equation?

Example 2. Solve the equations/initial value problems:

1. $xydx + (x+1)dy = 0$

2. $(x^2-1)y' + 2xy^2 = 0, \quad y(0) = 1$

3. $xydx - \sqrt{x^2+1}\ln^2 ydy = 0$

4. $x\cos^2 ydx - \mathrm{e}^x \sin 2ydy = 0, \quad y(0) = 0$