1. **Mixing.** A brine solution of salt flows at a constant rate of a L/min into a large tank that initially held A L of brine solution in which was dissolved b kg of salt. The solution inside the tank is kept well stirred and flows out of the tank at the rate c L/min. If the concentration of salt in the brine entering the tank is d kg/L, determine the mass of salt in the tank after t min.

Example 1. A tank initially contains 100 L of fresh water. A brine containing 200 g/L of salt salt flows into the tank at rate of 3 L/min. The solution inside the tank is kept well stirred and flows out of the tank at the rate 2 L/min. Determine the concentration of salt at any time.

2. **Compound interest.** Suppose that a sum of money S_0 is deposited in a bank or money fund that pays interest at an annual rate r. The value $S(t)$ of the investment at any time t depends on the frequency with which interest is compounded as well as on the interest rate. We assume that compounding takes place *continuously*. The rate of change of the value of the investment is dS/dt and it is equal to the interest rate r times the current value of the investment $S(t)$. Thus,

$$\frac{dS}{dt} = rS, \qquad S(0) = S_0$$

is the initial value problem that governs the process.

If we assume that the deposits or withdrawals take place at a constant rate k, then we need to replace the equation by

$$\frac{dS}{dt} = rS + k$$

Example 2. A home buyer can afford to spend no more than $800/month on mortgage payments. Suppose that the interest rate is 9% and that the term of the mortgage is 20 years. Assumed that interest is compounded continuously and that payments are also made continuously. Determine the maximum amount that this buyer can afford to borrow.

3. **Population models.** If we assume that the people die only of natural causes, we might expect the death rate also to be proportional to the size of the population. So,

$$\frac{dp}{dt} = k_1 p - k_2 p = (k_1 - k_2)p = kp,$$

where $k = k_1 - k_2$ and k_2 is the proportionality constant for the death rate. Let's assume that $k_1 > k_2$ so that $k > 0$. This gives the mathematical model

$$\frac{dp}{dt} = kp, \quad p(0) = p_0,$$

which is called the **Malthusian** or **exponential, law** of population growth.

We might assume that another component of the death is proportional to the number of two-party interactions. There are $p(p-1)/2$ such possible interactions for a population of size p. Thus, if we combine the birth rate with the death rate and rearrange constants, we get the **logistic model**

$$\frac{dp}{dt} = -Ap(p - p_1), \quad p(0) = p_0,$$

where $A = k_3/2$ and $p_1 = (2k_1/k_3) + 1$.

The function $p(t)$ is called the **logistic function**.

Example 3. The population of mosquitoes in a certain area increases at a rate proportional to the current population, and in the absence of other factors, the population doubles each week. There are 200,000 mosquitoes in the area initially and predators eat 20,000 mosquitoes/day. Determine the population of mosquitoes in the area at any time.

4

4. **Heating and cooling of buildings.** Let $T(t)$ represent the temperature inside the building at time t and view the building as a single compartment.

We will consider three main factors that affect the temperature inside the building. First is the heat produced by people, lights, and machines inside the building. This causes a rate of increase in temperature that we will denote by $H(t)$. Second is the heating (or cooling) supplied by the furnace (or air conditioner). This rate of increase (or decrease) in temperature will be represented by $U(t)$. The third factor is the effect of the outside temperature $M(t)$ on the temperature inside the building. Third factor can be modeled using **Newtons law of cooling**. This law states that

$$\frac{dT}{dt} = K(M(t) - T(t)).$$

The positive constant K depends on the physical properties of the building, K does not depend on M, T or t.

Summarizing, we have

$$\frac{dT}{dt} = K(M(t) - T(t)) + U(t) + H(t),$$

where $H(t) \geq 0$ and $U(t) > 0$ for furnace heating and $U(t) < 0$ for air conditioning cooling.

Example 4. An object with temperature 150^0 is placed in a freezer whose temperature is 30^0. Assume that the temperature of the freezer remains essentially constant. If the object is cooled to 120^0 after 8 min, what will its temperature be after 18 min?

5. **Falling body.** (See Chapter 1)

Example 5. A body of mass m is projected vertically upward with an initial velocity v_0 in a medium offering a resistance $k|v|$, where k is a constant. Assume that the gravitational attraction of the earth is constant.

(a) Find the velocity of the body at any time.

(b) Find $\lim\limits_{t \to 0} v(t)$