Section 2.4 **Differences between linear and nonlinear equations.**

- **Linear equations.**

  Theorem 1. Suppose $p(t)$ and $q(t)$ are continuous on some interval I that contains the point t_0. Then for any choice of initial value y_0, there exists a unique solution $y(t)$ on I to the initial value problem

  $$y' + p(t)y = q(t), \qquad y(t_0) = y_0$$

  Example 1. Determine (without solving the problem) an interval in which the solution of the given IVP is certain to exist:

  $$(t-3)y' + (\ln t)y = 2t$$

  1. $y(1) = 2$

  2. $y(5) = 6$

- **Nonlinear equations.**

  Theorem 2. Let the functions f and $\dfrac{\partial f}{\partial y}$ are continuous in some rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ containing the point (t_0, y_0). Then, in some interval $t_0 - h < t < t_0 + h$ contained $\alpha < t < \beta$, there is a unique solution of the initial value problem

  $$y' = f(t, y), \quad y(t_0) = y_0$$

Remarks:

1. By this theorem we can guarantee the existence of solution only for values of t which are suffiently closed to t_0, but not for all t.
2. Geometric consequence of the theorem is that two integral curves never intersect each other.
3. The condition "$\dfrac{\partial f}{\partial y}$ is continuous in some rectangle..." is important for uniqueness.

Example 2. Solve the initial value problem

$$y' = y^{1/3},\ y(0) = 0$$

Example 3. For the IVP

$$y' = \frac{\ln|ty|}{1 - t^2 + y^2}$$

state where in the ty-plane the hypotheses of Theorem 2 are satisfied.