

Section 2.5 Autonomous equations.

Definition. A first order differential equation of the form

$$\frac{dy}{dt} = f(y)$$

is called **autonomous** (the independent variable does not appear explicitly).

Definition. The constant solutions of the equation $\frac{dy}{dt} = f(y)$ are called **equilibrium solutions**. They correspond to the zeros of f . Zeros of f are called **critical points**.

Definition. An equilibrium solution is **stable** if any neighboring solution is attracted to it as t goes to infinity. An equilibrium solution is **unstable** if any neighboring solution is repelled by the solution. An equilibrium solution that is not stable or unstable is called **semistable**.

Let $y = y_0$ is an equilibrium solution. Then

- $y = y_0$ is **stable** if $f(y)$ changes sign at y_0 from "+" to "-".
- $y = y_0$ is **unstable** if $f(y)$ changes sign at y_0 from "-" to "+".
- $y = y_0$ is **semistable** if $f(y)$ does not change sign at y_0 .

For autonomous systems the slope on horizontal lines $y = y_0$ is the same and the qualitative analysis can be made on the so called **phase line portrait**.

Directions to draw phase line portrait:

1. Find all equilibrium points (i.e. roots of $f(y) = 0$), draw a horizontal line and mark those points on it.
2. Check the sign of $f(y)$ in each of the intervals determined by the equilibrium points. Over those intervals where $f(y) > 0$ draw arrows pointing to the right, and on those where $f(y) < 0$ draw arrows pointing to the left (indicating in which direction are solutions moving).

Example 1. For the equation $\frac{dy}{dt} = y^2 - 1$

1. Sketch the graph of f versus y .

