Section 2.6 **Exact equations and integrating factors.**

Given an equation

$$M(x, y)dx + N(x, y)dy = 0.$$

There exists an implicit solution of the equation $F(x, y) = C$ if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

$F(x, y)$ satisfies the following conditions:

$$\frac{\partial F(x, y)}{\partial x} = M(x, y), \qquad \frac{\partial F(x, y)}{\partial y} = N(x, y)$$

In such a case, the equation is called **exact**.

Example 1. Is the equation

$$(3x^2 - 2xy + 2)dx + (6y^2 - x^2 + 3)dy = 0$$

exact? If it is, solve it.

Example 2. Solve IVP:

$$3x^2 - y + (2y - x)y' = 0, \quad y(1) = 3$$

Example 3. Is the equation

$$x^2y^3 + x(1+y^2)y' = 0$$

exact?

Multiply the equation by the integrating factor $\mu(x, y) = \dfrac{1}{xy^3}$ and then solve it.

Given an equation

$$M(x,y)dx + N(x,y)dy = 0.$$

If $\dfrac{M_y - N_x}{N(x,y)}$ is a function of x only, then a solution $\mu(x)$ of the equation

$$\frac{d\mu}{dx} = \frac{M_y - N_x}{N(x,y)}\mu$$

is an integrating factor for the differential equation.

Example 4. Find an integrating factor for the equation

$$(3x^2y + 2xy + y^3)dx + (x^2 + y^2)dy = 0$$

and then solve the equation.