

## Brief table of Laplace transform

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}, \quad s > 0$
$e^{at}$	$\frac{1}{s-a}, \quad s > a$
$t^n, \quad n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
$\sin bt$	$\frac{b}{s^2 + b^2}, \quad s > 0$
$\cos bt$	$\frac{s}{s^2 + b^2}, \quad s > 0$
$e^{at}t^n, \quad n = 1, 2, \dots$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$\delta(t - t_0)$	$e^{-st_0}$

## Properties of Laplace transform

1.  $\mathcal{L}\{f + g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$
2.  $\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$  for any constant  $c$
3.  $\mathcal{L}\{e^{at}f(t)\} = F(s - a)$
4.  $\mathcal{L}\{f'\} = s\mathcal{L}\{f\} - f(0)$
5.  $\mathcal{L}\{f''\} = s^2\mathcal{L}\{f\} - sf(0) - f'(0)$
6.  $\mathcal{L}\{f^{(n)}\} = s^n\mathcal{L}\{f\} - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$
7.  $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f(t)\})$
8.  $\mathcal{L}\{u_c(t)f(t - c)\} = e^{-cs}\mathcal{L}\{f(t)\} = e^{-cs}F(s)$
9.  $u_c(t)f(t - c) = \mathcal{L}^{-1}\{e^{-cs}F(s)\}$ , where  $f(t) = \mathcal{L}^{-1}\{F(s)\}$

**Convolution integrals.** If  $G(s) = \mathcal{L}\{g(t)\}$  and  $H(s) = \mathcal{L}\{h(t)\}$  both exist for  $s > a \geq 0$ , and

$$F(s) = G(s)H(s), \quad s > a,$$

then

$$\mathcal{L}^{-1}\{F(s)\} = f(t) = \int_0^t g(t - \tau)h(\tau)d\tau = \int_0^t g(\tau)h(t - \tau)d\tau.$$