

**Due Tuesday, April 25 at the beginning of class.**

If you use convolutions, please write your answer in terms of convolution integrals.

1. Solve the initial value problem using the method of Laplace transform.

(a)  $y'' + 2y' + 2y = \delta(t - \pi)$ ,  $y(0) = 1$ ,  $y'(0) = 0$ ,

(b)  $y'' + 2y' + 3y = \sin t + \delta(t - 3\pi)$ ,  $y(0) = y'(0) = 0$

2. Express the solution of the initial value problem

$$4y'' + 4y' + 17y = g(t), \quad y(0) = y'(0) = 0$$

in terms of a convolution integral.

3. Transform the given equation/initial value problem into a system of first order equations.

(a)  $y'' + 0.5y' + 2y = 3 \sin t$

(b)  $y'' + 0.25y' + 4y = 2 \cos 3t$ ,  $y(0) = 1$ ,  $y'(0) = -2$ .

4. If  $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$  and  $B = \begin{pmatrix} i & 3 \\ 2 & 2i \end{pmatrix}$ , find

(a)  $3A - 2B$

(b)  $AB$

(c)  $BA$

5. If  $A = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}$ , find  $A^{-1}$ .

6. Find general solutions of the given system and sketch its phase portrait.

(a)  $\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \mathbf{x}$

(b)  $\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{x}$