

Due Thursday, Feb. 2 at the beginning of class.

1. Find the general solution of the equation. Do not forget to find constant solutions for separable equations, if any.

(a) $\frac{dy}{dx} = \frac{1-x^2}{y^2}$

(b) $(t+y+1)dt - dy = 0$

(c) $y^{-1}dy + ye^{\cos x} \sin x dx = 0$

(d) $(x^2+1)\frac{dy}{dx} + xy = x$

(e) $(x+xy^2)dx + e^{x^2}ydy = 0$

2. Solve the initial value problem:

(a) $\frac{dy}{dx} - \frac{y}{x} = xe^x, \quad y(1) = e - 1$

(b) $\frac{dy}{dx} = 2\sqrt{y+1} \cos x, \quad y(\pi) = 0$

(c) $t^3 \frac{dx}{dt} + 3t^2x = t, \quad x(2) = 0$

(d) $\sqrt{y} dx + (1+x)dy = 0, \quad y(0) = 1$

(e) $\sin x \frac{dy}{dx} + y \cos x = x \sin x \quad y\left(\frac{\pi}{2}\right) = 2$

3. Suppose a brine containing 0.2 kg of salt per liter runs into a tank initially filled with 500 L of water containing 5 kg of salt. The brine enters the tank at a rate of 5 L/min. The mixture, kept uniform by stirring, is flowing out at a rate of 5 L/min. Find a concentration, in kilograms per liter, of salt in the tank after 10 min.
4. An object of mass 8 kg is given an upward initial velocity of 20 m/sec and then allowed to fall under the influence of gravity. Assume that the force in newtons due to the air resistance is $16v$, where v is the velocity of the object in m/sec. Determine the equation of motion of the object. If the object is initially 100m above the ground, determine when the object will strike the ground.
5. Consider the initial value problem

$$ty' + 3y = 5t^2, \quad y(2) = 5.$$

Using MatLab, find the solution to the initial value problem.

Determine the behavior as t approaches 0 from the right and as t becomes large. This can be done by plotting the solution on intervals such as $0.5 \leq t \leq 5$ and $0.2 \leq t \leq 20$.