

Due Thursday, Feb. 9 at the beginning of class.

1. Determine (without solving the problem) an interval in which the solution of the given IVP is certain to exist:

$$(4 - t^2)y' + 2ty = 3t^2$$

(a)  $y(-3) = 1$

(b)  $y(0) = 2$

(c)  $y(3) = 0$

2. For the following equations state where in the  $ty$ -plane the hypotheses of Theorem 2 (section 2.4) are satisfied.

(a)  $y' = (1 - t^2 - y^2)$

(b)  $y' = \frac{y \cot t}{1 + y}$

3. Solve the following initial value problems and determine how the interval in which the solution exists depends on the initial value  $t_0$ .

(a)  $y' = \frac{-4}{t}y, \quad y(t_0) = y_0$

(b)  $y' + y^3 = 0 \quad y(t_0) = y_0.$

4. Verify that both  $y_1 = 1 - t$  and  $y_2 = \frac{-t^2}{4}$  are solutions to the same initial value problem

$$y'(t) = \frac{-t + (t^2 + 4y)^{(1/2)}}{2}, \quad y(2) = -1.$$

Does it contradict the existence and uniqueness theorem?

5. For each of the given equations find the equilibrium solutions, sketch the phase line and determine whether the equilibrium solutions are stable, unstable, or semistable, graph some solutions, if  $y(t)$  is the solution of the equation satisfying the initial condition  $y(0) = y_0$ , where  $-7 \leq y_0 \leq 7$ , find the limit of  $y(t)$  when  $t \rightarrow \infty$  and the limit of  $y(t)$  when  $t \rightarrow -\infty$ , solve the equation.

(a)  $\frac{dy}{dt} = 7y - y^2 - 10$

(b)  $\frac{dy}{dt} = y^3 - 2y^2 + y$