## Due Thursday, March 23 at the beginning of class.

1. Find the general solution of the differential equation using the method of variation of parameters.
(a) $y^{\prime \prime}+9 y=9 \sec ^{2}(3 t), 0<t<\pi / 6$
(b) $y^{\prime \prime}-2 y^{\prime}+y=\frac{e^{t}}{1+t^{2}}$
2. Verify that the given functions $y_{1}$ and $y_{2}$ satisfy the corresponding homogeneous equation; then use variation of parameters to find a particular solution of the given nonhomogeneous equation.
(a) $t^{2} y^{\prime \prime}-2 y=3 t^{2}-1, t>0, y_{1}(t)=t^{2}, y_{2}(t)=t^{-1}$
(b) $t y^{\prime \prime}-(1+t) y^{\prime}+y=t^{2} e^{2 t}, t>0, y_{1}(t)=1+t, y_{2}(t)=e^{t}$
3. A mass weighing 100 g stretches a spring 5 cm . If the mass is set in motion from its equilibrium position with a downward velocity of $10 \mathrm{~cm} / \mathrm{s}$, and if there is no dumping, determine the position of the mass at any time $t$. When does the mass first return to its equilibrium position?
4. A mass weighing 3 lb stretches a spring 3 in . If the mass is pushed upward, contracting the spring a distance of 1 in , and then set in the motion with a downward velocity of $2 \mathrm{ft} / \mathrm{s}$, and if there is no damping, find the the position $y$ of the mass at any time $t$. Determine the frequency, period, amplitude and phase of the motion.
5. A certain vibrating system satisfies the equation $y^{\prime \prime}+\gamma y^{\prime}+y=0$. Find the value of the damping coefficient $\gamma$ for which the quasi period of the damped motion is $50 \%$ greater than the period of the corresponding undamped motion.
