Due Thursday, March 30 at the beginning of class.

1. A mass weighting 8 lb is attached to a spring hanging from the ceiling and comes to rest at its equilibrium position. At $t = 0$, an external force $F(t) = 2\cos 2t$ lb is applied to the system. If the spring constant is 10 lb/ft and the damping constant is 1 lb-sec/ft, find the steady-state solution for the system. What is the resonance frequency for the system?

2. A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N. The mass is set in motion from its equilibrium position with an upward velocity of 10 m/s. Assuming that there is no damping and the mass is acted by an external force of $3\cos 5t$ N, find the steady-state solution of the system.

3. Use definition to find the Laplace transform of the given function.

   (a) $f(t) = t^2$

   (b) $f(t) = e^{6t}$

   (c) $f(t) = \begin{cases} e^t, & 0 < t < 1, \\ t + 1, & t > 1. \end{cases}$

   (d) $f(t) = \begin{cases} t^2, & 0 \le t \le 1 \\ 2 + t, & 1 < t \le 2, \\ 6 - t, & t > 2. \end{cases}$

   (e) $f(t) = \begin{cases} 2, & 0 \le t \le 1 \\ 1 - t, & 1 < t \le \dfrac{\pi}{2}, \\ \sin t, & t > \dfrac{\pi}{2}. \end{cases}$

4. Use the table and properties of Laplace transform to determine the following transforms.

   (a) $\mathcal{L}\{t^3 - te^t + e^{4t}\cos t\}$

   (b) $\mathcal{L}\{t\cos t\}$

   (c) $\mathcal{L}\{e^{-2t}\sin 2t + e^{3t}t^2\}$

   (d) $\mathcal{L}\{te^{at}\sin bt\}$

   (e) $\mathcal{L}\{te^{at}\cos bt\}$