MATH 308 **Practice Problems for Test 1** **Summer 2018**

over sections 1.1–1.3, 2.1–2.6, 3.1 – 3.5

1. A large tank initially contains 10 L of fresh water. A brine containing 20 g/L of salt flows into the tank at a rate of 3 L/min. The solution inside the tank is kept well stirred and flows out of the tank at a rate of 2 L/min. Determine the concentration of salt in the tank as a function of time.

2. An object with temperature 150° is placed in a freezer whose temperature is 30°. Assume that the temperature of the freezer remains essentially constant.

   (a) If the object is cooled to 120° after 8 min, what will its temperature be after 18 min?

   (b) When will its temperature be 60°?

3. Determine (without solving the problem) an interval in which the solution to the initial value problem

$$(4 - t^2)y' + 2ty = 3t^2, \qquad y(1) = -3$$

   is certain to exist.

4. Solve the initial value problem

$$y' = \frac{t^2}{1 + t^3}, \qquad y(0) = y_0$$

   and determine how the interval in which the solution exists depends on the initial value y_0.

5. Solve the following initial value problem

$$\sqrt{y}dt + (1 + t)dy = 0 \quad y(0) = 1.$$

6. Find the general solution of the equation

$$(t^2 - 1)y' + 2ty + 3 = 0$$

7. Given the differential equation

$$\frac{dy}{dt} = 7y - y^2 - 10$$

   (a) Find the equilibrium solutions

   (b) Sketch the phase line and determine whether the equilibrium solutions are stable, unstable, or semistable

   (c) Graph some solutions

   (d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, find the limit of $y(t)$ when $t \to \infty$

   (e) Solve the equation

8. Solve the initial value problem

$$(ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x)dx + (xe^{xy}\cos(2x) - 3)dy = 0, \qquad y(0) = -1$$

9. Find an integrating factor for the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

   and then solve the equation.

10. Solve the equation/initial value problem

    (a) $6y'' - 5y' + y = 0$, $y(0) = 4$, $y'(0) = 0$

    (b) $4y'' - 12y' + 9y = 0$

    (c) $y'' + 4y' + 5y = 0$, $y(0) = 0$, $y'(0) = 1$

11. Find the interval(s) on which the solution of the initial value problem

$$x^3 y'' + \frac{x}{\sin x} y' - \frac{2}{x-5} y = 0, \quad y(2) = 6, \quad y'(2) = 7$$

    is certain to exist.

12. Find the Wronskian of two functions $y_1(x) = x + 2x^2$ and $y_2(x) = 2^x$.