Math 308 – 201 **Quiz 1** 7/5/2018

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer

1. Determine the value for r for which the differential equation

$$t^2y'' + 4ty' + 2y = 0$$

has solutions of the form $y = t^r$ for $t > 0$.

$y = t^r$

$y' = rt^{r-1}$ 1pt

$y'' = r(r-1)t^{r-2}$ 1pt

$t^2[r(r-1)t^{r-2}] + 4rt^{r-1}\cdot t + 2 = 0$ 2pts

$r(r-1)t^r + 4rt^r + 2t^r = 0$ 2pts

$r(r-1) + 4r + 2 = 0$ 1pt

$r^2 - r + 4r + 2 = 0$

$r^2 + 3r + 2 = 0$ 1pt

$\boxed{r_1 = -1, \quad r_2 = -2}$ 2pts.

correct answer −10 pts.

Math 308 – 201 Quiz 2 7/6/2018

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer

1. Find the general solution of the equation
$$y' = \frac{3x^2 - 1}{3 + 2y}.$$ 3pts

2. Solve the initial value problem
$$y' - 2y = e^{2t}, \quad y(0) = 2.$$ 7 pts

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1. $\frac{dy}{dx} = \frac{3x^2-1}{3+2y}$

$(3+2y)dy = (3x^2-1)dx$ 1pt

$\boxed{3y + y^2 = x^3 - x + c}$ 1pt

1 pt.

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2. $y' - 2y = e^{2t}$, $y(0) = 2$.

Integrating factor $\mu(t)$: $\frac{d\mu}{dt} = -2\mu$

$\frac{d\mu}{\mu} = -2dt$

$\ln|\mu| = -2t + \cancel{c}^{\,0}$

3pts $\mu(t) = e^{-2t}$

$\frac{d}{dt}(ye^{-2t}) = e^{2t}e^{-2t}$

$ye^{-2t} = t + C$

$y(t) = te^{2t} + Ce^{2t}$

general solution

3 pts

$y(0) = C = 2 \Rightarrow$ $\boxed{y(t) = te^{2t} + 2e^{2t}}$ 1pt

Math 308 – 201 Quiz 3 7/9/2018

NAME (print): key

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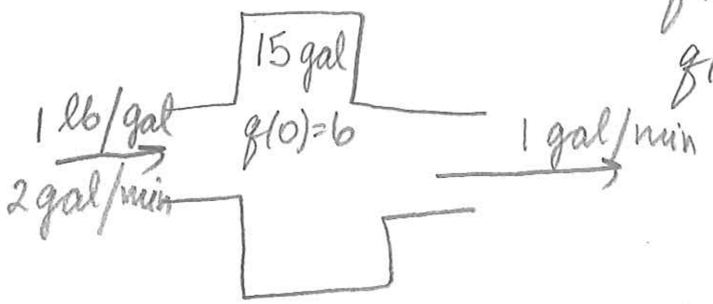
1. A 30-gallon tank initially contains 15 gallons of salt water containing 6 pounds of salt. Suppose salt water containing 1 pound of salt per gallon is pumped into the tank at a rate of 2 gallons per minute, while a well-mixed solution leaves the tank at a rate of 1 gallon per minute. How much salt is in the tank at time t?



$q(t)$ mass of salt @ time t

$q(0)=6$

$$\frac{dq}{dt} = \boxed{\text{rate in}} - \boxed{\text{rate out}}$$

rate in $= (1)(2) = 2$

rate out $= \dfrac{q(t)}{15+(2-1)t} \cdot 1 = \dfrac{q}{15+t}$

$$\frac{dq}{dt} = 2 - \frac{q}{15+t}, \quad q(0) = 6$$

$$\frac{dq}{dt} + \frac{q}{15+t} = 2 \quad \text{— linear}$$

integrating factor $\mu(t)$: $\dfrac{d\mu}{dt} = \dfrac{\mu}{15+t}$

$$\frac{d\mu}{\mu} = \frac{dt}{15+t} \Rightarrow \ln|\mu| = \ln|15+t|$$

$$\mu(t) = 15+t$$

$$(15+t)\,q(t) = \int 2(15+t)\,dt$$

$$(15+t)\,q(t) = (15+t)^2 + C$$

$$q(t) = (15+t) + \frac{C}{15+t}$$

$$q(0) = 15 + \frac{C}{15} = 6 \Rightarrow \frac{C}{15} = -9 \Rightarrow C = 135$$

$$\boxed{q(t) = 15+t - \frac{135}{15+t}}$$

Math 308 – 201 Quiz 4 7/10/2018

NAME (print): key

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1. Find the interval on which the solution of the initial value problem

$$y' \ln t + y = \cot t, \quad y(2) = 3$$

is certain to exist.

2. For the equation

$$\frac{dy}{dt} = y^2 + y - 6$$

(a) Find the equilibrium solutions

(b) Sketch the phase line and determine whether the equilibrium solutions are stable, unstable, or semistable

3. Is the equation

$$\left(\frac{y}{x} + 6x\right) + (y \ln x + xy)y' = 0$$

exact?

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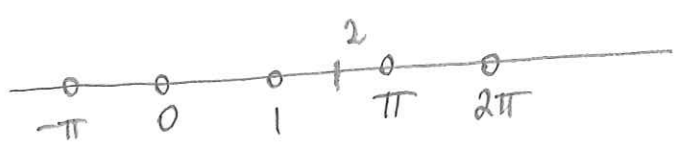
1. $y' + \frac{y}{\ln t} = \frac{\cot t}{\ln t}$, $y(2) = 3$.

continuous whenever $t > 0$, $t \neq 1$ $(\ln t \neq 0)$

$t \neq 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$



$\boxed{(1, \pi)}$

2. a) $y^2 + y - 6 = 0$

$(y+3)(y-2) = 0 \Rightarrow y_1 = -3, \; y_2 = 2$

b)

$f(0) = -6 < 0$

$f(3) = 9 + 3 - 6 > 0$

$f(-4) = 16 - 4 - 6 > 0$

$\boxed{\begin{array}{l} y = -3 \text{ stable} \\ y = 2 \text{ unstable} \end{array}}$

3. $\underbrace{\left(\frac{y}{x}+6x\right)}_{M}+\underbrace{(y\ln x+xy)}_{N}y'=0$

$\frac{\partial M}{\partial y}=\frac{1}{x}, \quad \frac{\partial N}{\partial x}=\frac{y}{x}+y$

don't match, $\boxed{\text{not exact}}$

Math 308 – 201 Quiz 5 7/11/2018

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer

1. Find the general solution of the equation $(2xy^2 + 2y) + (2x^2y + 2x)y' = 0$
2. Find the longest interval in which the solution of the initial value problem

$$(t^2 - 2t)y'' + ty' - (t+2)y = 0, \quad y(1) = 1, y'(1) = 0$$

is certain to exist.

3. Find the Wronskian of $f(t) = \cos(2t)$ and $g(t) = \sin(2t)$.

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1. $(2xy^2+2y)+(2x^2y+2x)y'=0$

$\underbrace{(2xy^2+2y)}_{M}dx + \underbrace{(2x^2y+2x)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = 4xy+2 \qquad \frac{\partial N}{\partial x} = 4xy+2$ match, exact

$F(x,y): \begin{cases} \frac{\partial F}{\partial x} = 2xy^2+2y \\ \frac{\partial F}{\partial y} = 2x^2y+2x \end{cases}$ $\quad \int \frac{\partial F}{\partial x}dx = \int [2xy^2+2y]dx$

$F(x,y) = x^2y^2+2xy+g(y)$

$\frac{\partial F}{\partial y} = 2x^2y + 2x + g'(y) = 2x^2y+2x \Rightarrow g'(y)=0 \Rightarrow g(y)=C$

$F(x,y) = x^2y^2+2xy+C$

General solution: $\boxed{x^2y^2+2xy+C=0}$

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2. $y'' + \frac{t}{t^2-2t}y' - \frac{t+2}{t^2-2t}y = 0$, $y(1)=1, y'(1)=0$

continuous whenever $t \neq 0$, $t \neq 2$

$\boxed{(0,2)}$

3. $W[\cos 2t, \sin 2t] = \begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix}$

$= 2\cos^2 2t + 2\sin^2 2t = 2(\underbrace{\cos^2 2t + \sin^2 2t}_{1}) = \boxed{2}$

Math 308 – 201 Quiz 6 7/12/2018

NAME (print): key

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1. Find the general solution of the equation:

   (a) $2y'' - 7y' + 3y = 0$
   (b) $y'' - y = 0$
   (c) $y'' - 2y' + y = 0$
   (d) $4y'' + 4y' + y = 0$
   (e) $y'' + y = 0$
   (f) $y'' + 6y' + 13y = 0$

(a) $2y'' - 7y' + 3y = 0$

$2r^2 - 7r + 3 = 0$

$r_1 = \frac{7 + \sqrt{49 - 24}}{4} = \frac{7+5}{4} = 3$

$r_2 = \frac{7-5}{4} = \frac{1}{2}$

$\boxed{y(t) = C_1 e^{3t} + C_2 e^{\frac{t}{2}}}$

(b) $y'' - y = 0$

$r^2 - 1 = 0$

$r = \pm 1$

$\boxed{y(t) = C_1 e^{t} + C_2 e^{-t}}$

(c) $y'' - 2y' + y = 0$

$r^2 - 2r + 1 = 0$

$(r-1)^2 = 1 \Rightarrow r = 1$ repeated

$\boxed{y(t) = (C_1 + C_2 t)e^{t}}$

(d) $4y'' + 4y' + y = 0$

$4r^2 + 4r + 1 = 0$

$(2r+1)^2 = 0 \Rightarrow r = -\frac{1}{2}$ repeated

$\boxed{y(t) = (C_1 + C_2 t)e^{-\frac{t}{2}}}$

(e) $y'' + y = 0$

$r^2 + 1 = 0 \Rightarrow r = \pm i$

$Re(r) = 0,\ Im(r) = 1$

$y(t) = (C_1 \cos t + C_2 \sin t)e^{0 \cdot t}$

$\boxed{y(t) = C_1 \cos t + C_2 \sin t}$

(f) $y'' + 6y' + 13y = 0$.

$r^2 + 6r + 13 = 0$

$r_1 = \frac{-6 + \sqrt{36 - 52}}{2} = \frac{-6 + \sqrt{-16}}{2}$

$r_1 = -3 + 2i$

$Re(r_1) = -3,\ Im(r_1) = 2$

$\boxed{y(t) = e^{-3t}(C_1 \cos 2t + C_2 \sin 2t)}$