Math 308 – 200 Quiz 7 7/18/2018

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer

1. Find the form for a particular solution $y_p(t)$ for the equation

$$y'' - 4y' + 4y = g(t)$$

$r^2 - 4r + 4 = 0$, $r = 2$ repeated

$y_h(t) = (C_1 + C_2 t)e^{2t}$

if

2pts for (a)

(a) $g(t) = t + 3 - 16te^{2t} + 8t^2e^{-t}$

$y_p(t) = At + B + (Ct + D)t^2 e^{2t} + (Et^2 + Ft + G)e^{-t}$

(b) $g(t) = 3\sin t - t\cos 2t$

$y_p(t) = A\sin t + B\cos t + (Ct + D)\cos 2t + (Et + F)\sin 2t$

2pts. for (b)

2. Use the variation of parameters to find the general solution of $y'' + y = \sec x$

$y'' + y = 0$, $r^2 + 1 = 0$, $r = \pm i$

$y_h(x) = C_1 \cos x + C_2 \sin x$. 2 points

$y(x) = C_1(x)\cos x + C_2(x)\sin x$ 1pt

$W[\cos x, \sin x] = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$ 1point

$C_1(x) = -\int \sec x \cdot \sin x \, dx$ 1 point

$C_2(x) = \int \sec x \cdot \cos x \, dx$ 1 pt

Math 308 – 200 Quiz 8 7/19/2018

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer

1. A mass weighing 4 lb stretches a spring 1.5 in. The mass is given a positive displacement 2 in from its equilibrium position and released with no initial velocity. Assuming that there is no damping and the mass is acted on by an external force of $2\cos 3t$ lb,

   (a) Formulate the initial value problem describing the motion of mass

   (b) Solve the initial value problem. [redacted]

   (c) If the given external force is replaced by a force $4\cos\omega t$ of frequency ω, find the value of ω for which resonance occurs.

1pt

$$m = \frac{4}{32} = \frac{1}{8}, \quad \frac{1.5}{12}k = 4 \Rightarrow k = \frac{4 \cdot 12}{1.5} = \frac{4 \cdot 12 \cdot 2}{3} = 32 \quad \text{1pt}$$

$$\frac{1}{8}u'' + 32u = 2\cos 3t \quad \text{1pt}$$

1pt 1pt

$$\text{1pt} \quad \boxed{u'' + 256u = 16\cos 3t, \quad u(0) = \frac{1}{6}, \quad u'(0) = 0}$$

$$u'' + 256u = 0 \Rightarrow u_h(t) = C_1\cos 16t + C_2\sin 16t \quad \text{1pt}$$

$$u_p = A\cos 3t + B\sin 3t \quad \text{1pt}$$

$$u_p'' = -9A\cos 3t - 9B\sin 3t$$

$$-9A\cos 3t - 9B\sin 3t + 256A\cos 3t + 256B\sin 3t = 16\cos 3t$$

$$\boxed{B = 0} \text{ 1pt}, \quad \boxed{A = 16/247} \text{ 1pt}$$

$$u_p = \frac{16}{247}\cos 3t$$

$$u = C_1\cos 16t + C_2\sin 16t + \frac{16}{247}\cos 3t$$

$$u(0) = C_1 + \frac{16}{247} = \frac{1}{6}, \quad C_1 = \frac{1}{6} - \frac{16}{247} \quad \text{1pt}$$

$$u'(0) = 16C_2 = 0 \Rightarrow C_2 = 0.$$

$$\boxed{u = \left(\frac{1}{6} - \frac{16}{247}\right)\cos 16t + \frac{16}{247}\cos 3t}$$

$F_{ext.} = 4\cos\omega t$, $\boxed{\omega = 16,}$ then the system is in resonance.

1pt.

Math 308 – 200 Quiz 9 7/20/2018

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer

Brief table of Laplace transform

$f(t) = \mathcal{L}^{-1}\{f\}(s)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}, \quad s > 0$
e^{at}	$\frac{1}{s-a}, \quad s > a$
$t^n, \quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
$\sin bt$	$\frac{b}{s^2+b^2}, \quad s > 0$
$\cos bt$	$\frac{s}{s^2+b^2}, \quad s > 0$
$e^{at}t^n, \quad n = 1, 2, \ldots$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
$e^{ct} f(t)$	$F(s-c)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{(n-1)}(0)$

1. Find the Laplace transform of the function $f(t) = 2t^2 e^{-t} - t + t\cos t$.

2. Find the inverce Laplace transform of the function $F(s) = \dfrac{s+3}{s^2+4s+5}$.

1. $\mathcal{L}\{2t^2 e^{-t} - t + t\cos t\} = 2 \cdot \frac{2}{(s+1)^3} - \frac{1}{s^2} + (-1)\left(\frac{s}{s^2+1}\right)'$

$= \frac{4}{(s+1)^3} - \frac{1}{s^2} - \frac{s^2+1-2s^2}{(s^2+1)^2} = \boxed{\frac{4}{(s+1)^3} - \frac{1}{s^2} - \frac{1-s^2}{(s^2+1)^2}}$

2. $\frac{s+3}{(s^2+4s+5)} = \frac{s+3}{(s+2)^2+1} = \frac{s+2}{(s+2)^2+1} + \frac{1}{(s+2)^2+1}$

$\mathcal{L}\{-\} = \boxed{e^{-2t}\cos t + e^{-2t}\sin t}$

Math 308 – 201 Quiz 10 7/23/2018

NAME (print): key

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Brief table of Laplace transform

$f(t) = \mathcal{L}^{-1}\{f\}(s)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}$, $s > 0$
e^{at}	$\frac{1}{s-a}$, $s > a$
t^n, $n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$, $s > 0$
$\sin bt$	$\frac{b}{s^2+b^2}$, $s > 0$
$\cos bt$	$\frac{s}{s^2+b^2}$, $s > 0$
$e^{at}t^n$, $n = 1, 2, \ldots$	$\frac{n!}{(s-a)^{n+1}}$, $s > a$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$, $s > a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$, $s > a$
$e^{ct}f(t)$	$F(s-c)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$

1. Solve the initial value problem

$$y'' + 2y' - 3y = 1, \qquad y(0) = 0, y'(0) = 1$$

using the Laplace Transform.

$\mathcal{L}\{y(t)\} = Y(s)$

$\mathcal{L}\{y'(t)\} = sY(s) - y(0) = sY(s)$ 1pt

$\mathcal{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - 1$ 1pt

$s^2Y(s) - 1 + 2sY(s) - 3Y(s) = \frac{1}{s}$ 1pt

$(s^2+2s-3)Y(s) = \frac{1}{s} + 1 = \frac{s+1}{s}$

$Y(s) = \frac{s+1}{(s^2+2s-3)s}$ 1pt $= \frac{s+1}{s(s+3)(s-1)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s-1}$

$A = -1/3$ 1pt, $B = -1/6$ 1pt, $C = +1/2$ 1pt

$\boxed{y(t) = \mathcal{L}^{-1}\{Y(s)\} = -\frac{1}{3} - \frac{1}{6}e^{-3t} + \frac{1}{2}e^{t}}$ 3pts (final answer)

Math 308 – 201 Quiz 11 7/24/2018

NAME (print): ______________________

No credit for unsupported answers will be given. Clearly indicate your final answer

1. Find the Laplace transform of the function

   (a) $f(t) = \begin{cases} \frac{t}{2}, & 0 \le t < 6 \\ 3, & t \ge 6 \end{cases}$

   (b) $f(t) = (t^2 - 2t + 2)u_1(t)$

   (c) $f(t) = e^t \delta(t-1)$

2. Find the inverse Laplace transform of the function

   (a) $F(s) = \dfrac{e^{-3s}}{s^4}$

   (b) $F(s) = \dfrac{(s+3)e^{-2s}}{s^2+4s+5}$ $\mathcal{L}^{-1} = u_2(t)\left[e^{-2(t-2)}\cos(t-2) + e^{-2(t-2)}\sin(t-2)\right]$ 1pt

1 (a). $f(t) = \frac{t}{2} + \left(3 - \frac{t}{2}\right)u_6(t)$ 1pt

$= \frac{t}{2} + \frac{6-t}{2}u_6(t) = \frac{t}{2} - \frac{1}{2}(t-6)u_6(t)$

$\frac{1}{2s^2}(1-e^{-6s}) = \boxed{\mathcal{L}\{f\} = \frac{1}{2}\cdot\frac{1}{s^2} - \frac{1}{2}\frac{1}{s^2}e^{-6s}}$ 2pts

(1b) $f(t) = (t^2 - 2t + 1 + 1)u_1(t) = (t-1)^2 u_1(t) + u_1(t)$

$\mathcal{L}\{f\} = \mathcal{L}\{t^2\}e^{-s} + \frac{e^{-s}}{s} = \boxed{\frac{2}{s^3}e^{-s} + \frac{e^{-s}}{s}}$ 2pts

(1c) $\mathcal{L}\{\delta(t-1)\} = e^{-s}$

$\mathcal{L}\{e^t\delta(t-1)\} = \boxed{e^{-(s-1)}}$ 1pt

(2a) $\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^4}\right\} = \boxed{\frac{1}{6}(t-3)^3 u_3(t)}$ 1pt

$\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} = \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{6}{s^4}\right\} = \frac{1}{6}t^3$ 1pt

(2b) $\mathcal{L}^{-1}\left\{\frac{s+3}{s^2+4s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{(s+2)+1}{(s+2)^2+1}\right\} = e^{-2t}\cos t + e^{-2t}\sin t$ 1pt

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NAME (print): ______________________

No credit for unsupported answers will be given. Clearly indicate your final answer

Brief table of Laplace transform

$f(t) = \mathcal{L}^{-1}\{f\}(s)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}, \quad s > 0$
e^{at}	$\frac{1}{s-a}, \quad s > a$
$t^n, \quad n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
$\sin bt$	$\frac{b}{s^2+b^2}, \quad s > 0$
$\cos bt$	$\frac{s}{s^2+b^2}, \quad s > 0$
$e^{at}t^n, \quad n = 1, 2, \dots$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{ct}f(t)$	$F(s-c)\ u_c(t)$
$\delta(t-t_0)$	e^{-st_0}
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$

1. Find the Laplace transform of the function $f(t) = \int_0^t (t-\tau)e^{3\tau}d\tau$.

2. Find the inverse Laplace transform of the function $F(s) = \frac{s}{(s^2+9)(s+4)^3}$.

3. Transform the initial value problem

$$e^t y'' + t^2 y' - \sin t\, y = 3\arctan t, \qquad y(0) = 5, \quad y'(0) = 3$$

into a system of first order equation, then in matrix notation.

1. $f(t) = \int_0^t (t-\tau)e^{3\tau}d\tau$

$h = t-\tau \Rightarrow h(t) = t$

$H(s) = \frac{1}{s^2}$ 1

$g(\tau) = e^{3\tau}, \ g(t) = e^{3t}$

$G(s) = \frac{1}{s-3}$ 1

$F(s) = \frac{1}{s^2(s-3)}$ 1

2. $\frac{s}{(s^2+9)(s+4)^3}$

$= \frac{s}{s^2+9} \cdot \frac{1}{(s+4)^3}$ 1

$\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} = \cos 3t$ 1

$\mathcal{L}^{-1}\left\{\frac{1}{(s+4)^3}\right\} = \frac{1}{2}t^2e^{-4t}$ 1

$\mathcal{L}^{-1}\{\ \} = \frac{1}{2}\int_0^t (t-\tau)^2 e^{-4(t-\tau)}\cos 3\tau\, d\tau$

$= \frac{1}{2}\int_0^t t^2 e^{-4t}\cos 3(t-\tau)\,d\tau$

1

$y = x_1$

$y' = x_2$

$y'' = x_2' = \frac{3\arctan t}{e^t} + \frac{\sin t}{e^t}y - \frac{t^2}{e^t}y'$ ~~3arctan~~

more on back

$\begin{cases} x_1' = x_2 \\ x_2' = (\sin t)e^{-t}x_1(t) - t^2e^{-t}x_2(t) + 3e^{-t}\arctan t \end{cases}$ 2pts.

1pt

$x_1(0) = 5$

$x_2(0) = 3$