Chapter 2. First Order Differential Equations Section 2.2 Separable Equations

$$\frac{dy}{dx} = f(x, y)$$

Sometimes a function f(x, y) can be represented as a product of two functions, one of which depens ONLY on x, another depens ONLY on y, or f(x, y) = g(x)h(y). Then

$$\frac{dy}{dx} = g(x)h(y).$$

Definition. A differential equation y' = f(x, y) is called **separable** if it can be written in the form

M(x)dx + N(y)dy = 0

Example 1. Determine whether the given equation is separable.

- 1. $(t-2y)^2y'=2$
- 2. $y^4 e^y + (t^3 + 1)y' = y'(t^3 + 1)e^{2y}$
- 3. $yx \ln x dx \sqrt{y} dy + x \ln x dx = 0$
- 4. $y' = \cot^2\left(\frac{x}{2} + y 1\right) + \frac{1}{2}$

How to solve a separable equation?

Example 2. Solve the equations/initial value problems:

1. xydx + (x+1)dy = 0

2.
$$(x^2 - 1)y' + 2xy^2 = 0$$
, $y(0) = 1$

3.
$$xydx - \sqrt{x^2 + 1} \ln^2 ydy = 0$$

4. $x \cos^2 y dx - e^x \sin 2y dy = 0$, y(0) = 0