Section 3.1 **Homogeneous Equations with Constant Coefficients**

We begin our discussion with homogeneous equations with constant coefficients

$$ay'' + by' + cy = 0$$

where a, b, c are constants.

We try to find a solution of the form $y = \mathrm{e}^{rt}$.

Definition. An equation

$$ar^2 + br + c = 0$$

is called the **auxiliary equation** or **characteristic equation** associated with equation $ay'' + by' + cy = 0$.

If $b^2 - 4ac > 0$, then the auxiliary equation has two distinct real roots r_1 and r_2. Then

$$y(x) = c_1\mathrm{e}^{r_1 t} + c_2\mathrm{e}^{r_2 t}$$

is the general solution to the equation.

Example 1. Find the general solution to the given equation

(a) $y'' - y' - 2y = 0$.

(b) $y'' + 7y' + 10y = 0$.

Example 2. Solve the given initial value problems.

(a) $y'' + y' = 0$, $y(0) = 2$, $y'(0) = 1$.

(b) $y'' + 4y' - 5y = 0$, $y(0) = 11$, $y'(0) = -7$.