Section 3.4 **Repeated roots; reduction of order.**

$$ay'' + by' + cy = 0$$

where a, b, c are constants and $b^2 - 4ac = 0$.

Then the auxiliary equation

$$ar^2 + br + c = 0$$

has one repeated root

$$r = -\frac{b}{2a}$$

and the corresponding solution of the equation is $y_1(t) = e^{-\frac{b}{2a}t}$.

We use **reduction of order** to find a second solution.

We'll look for the general solution of the form

$$y(t) = v(t)e^{-\frac{b}{2a}t},$$

where $v(t)$ is an unknown function. Let's plug $y(t)$ back into the equation:

If the auxiliary equation $ar^2 + br + cr = 0$ has one repeated root

$$r = -\frac{b}{2a},$$

then the general solution of the equation $ay'' + by' + cy = 0$ is

$$y(t) = e^{-\frac{b}{2a}t}\left(C_1 + C_2 t\right),$$

where C_1 and C_2 are arbitrary constants.

Example 1. Find the general solution of the equation

$$9y'' + 6y' + y = 0$$

Example 2. Solve the initial value problem

$$y'' - 4y' + 4y = 0, \qquad y(0) = 0, y'(0) = 2.$$