Section 6.2 **Solution of initial value problems.**

To solve an initial value problem:

- Take the Laplace transform of both sides of the equation.
- Use the properties of the Laplace transform and the initial conditions to obtain an equation for the Laplace transform of the solution and then solve this equation for the transform.
- Determine the inverse Laplace transform of the solution.

Important formulas:

$$\mathcal{L}\{y'\}(s) = s\mathcal{L}\{y\}(s) - y(0)$$
$$\mathcal{L}\{y''\}(s) = s^2\mathcal{L}\{y\}(s) - sy(0) - y'(0)$$

Example 1. Solve the initial value problem.

1. $y'' + 6y' + 9y = 0$, $y(0) = -1$, $y'(0) = 6$

2. $y'' + 6y' + 5y = 12e^t$, $y(0) = -1$, $y'(0) = 7$.

3. $y'' - 2y' + 2y = e^{-t}$, $y(0) = 0$, $y'(0) = 1$.