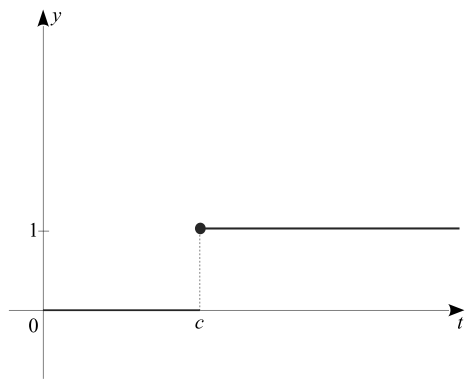
Section 6.3 **Step functions.**

Definition. The **unit step function** or **Heaviside function** is defined by

$$u_c(t) = \begin{cases} 0, & t < c, \\ 1, & t \geq c, \end{cases} \qquad c \geq 0.$$



Graph of $y = u_c(t)$

Example 1. Express $f(t)$ in terms of $u_c(t)$ if

1. $f(t) = \begin{cases} 0, & 0 \leq t < 3, \\ -2, & 3 \leq t < 5, \\ 2, & 5 \leq t < 7, \\ 1, & t \geq 7. \end{cases}$

2. $f(t) = \begin{cases} t, & 0 \leq t < 1, \\ t-1, & 1 \leq t < 2, \\ t-2, & 2 \leq t < 3, \\ 0, & t \geq 3. \end{cases}$

$$\boxed{\mathcal{L}\{u_c(t)\} = \frac{e^{-cs}}{s}, \qquad s > 0.}$$

Theorem. If $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > a \geq 0$, and c is a positive constant, then

$$\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\} = e^{-cs}F(s), \qquad s > a.$$

Conversely, if $f(t) = \mathcal{L}^{-1}\{F(s)\}$, then

$$u_c(t)f(t-c) = \mathcal{L}^{-1}\{e^{-cs}F(s)\}$$

Example 2. Find the Laplace transform of the following functions:

1. $f(t) = u_1(t) + 2u_3(t) - 6u_4(t)$

2. $f(t) = (t-3)u_2(t) - (t-2)u_3(t)$

3. $f(t) = \begin{cases} 0, & t < 1, \\ t^2 - 2t + 2, & t \geq 1, \end{cases}$

Example 3. Find the inverse Laplace transform of the given function.

1. $F(s) = \dfrac{e^{-2s}}{s^2 + s - 2}$

2. $F(s) = \dfrac{2(s-1)e^{-2s}}{s^2 - 2s + 2}$