Section 6.4 Differential equations with discontinuous forcing functions.

Important formulas:

1. $\mathcal{L}\{y'\}(s) = s\mathcal{L}\{y\}(s) - y(0)$

2. $\mathcal{L}\{y''\}(s) = s^2\mathcal{L}\{y\}(s) - sy(0) - y'(0)$

3. $\mathcal{L}\{u_c(t)\} = \dfrac{e^{-cs}}{s}$

4. $\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}$

5. $\mathcal{L}^{-1}\{e^{-cs}F(s)\} = u_c(t)f(t-c)$, where $F(s) = \mathcal{L}\{f(t)\}$

Example 1. Solve the initial value problem.

$$y'' + 5y' + 6y = g(t), \qquad y(0) = 0, y'(0) = 2,$$

where $g(t) = \begin{cases} 0, & 0 \le t < 1, \\ t, & 1 < t < 5, \\ 1, & 5 < t. \end{cases}$