

Section 6.5 Impulse Functions.

**Definition.** A **unit impulse function** (**Dirac delta function**) is a function defined by

$$\delta(t) = \begin{cases} 1, & t = 0, \\ 0, & t \neq 0. \end{cases}$$

**Property of the unit impulse function:**

$$\int_{-\infty}^{\infty} \delta(t) dt = 1.$$

$$\mathcal{L}\{\delta(t - t_0)\} = e^{-st_0}$$

**Example 1.** Solve the initial value problem.

$$y'' + 2y' + 2y = \delta(t - \pi), \quad y(0) = 1, y'(0) = 0.$$